Math 206 Complex Calculus
Quiz-1
Solutions

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1) Find all the cube roots of **(a)** i, **(b)** $-i$.

Solution a: Let $\theta_0 = \text{Arg}(i)$. Then $\theta_0 = \dfrac{\pi}{2} = 90°$.

$\dfrac{\theta_0}{3} = \dfrac{\pi}{6} = 30°$, so $c_0 = \cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6} = \dfrac{\sqrt{3}}{2} + i\dfrac{1}{2}$.

$\dfrac{\theta_0}{3} + \dfrac{2\pi}{3} = \dfrac{5\pi}{6} = 150° = 180° - 30°$, so $c_1 = \cos\dfrac{5\pi}{6} + i\sin\dfrac{5\pi}{6} = -\dfrac{\sqrt{3}}{2} + i\dfrac{1}{2}$.

$\dfrac{\theta_0}{3} + \dfrac{4\pi}{3} = \dfrac{3\pi}{2} = 270° = 180° + 90°$, so $c_2 = \cos\dfrac{3\pi}{2} + i\sin\dfrac{3\pi}{2} = -i$.

Thus c_0, c_1, c_2 are all the cube roots of i, given in rectangular form.

Solution b: Let $\theta_0 = \text{Arg}(-i)$. Then $\theta_0 = -\dfrac{\pi}{2} = -90°$.

$\dfrac{\theta_0}{3} = -\dfrac{\pi}{6} = -30°$, so $c_0 = \cos\dfrac{-\pi}{6} + i\sin\dfrac{-\pi}{6} = \dfrac{\sqrt{3}}{2} - i\dfrac{1}{2}$.

$\dfrac{\theta_0}{3} + \dfrac{2\pi}{3} = \dfrac{\pi}{2} = 90°$, so $c_1 = \cos\dfrac{\pi}{2} + i\sin\dfrac{\pi}{2} = i$.

$\dfrac{\theta_0}{3} + \dfrac{4\pi}{3} = \dfrac{7\pi}{6} = 210° = 180° + 30°$ so $c_2 = \cos\dfrac{7\pi}{6} + i\sin\dfrac{7\pi}{6} = -\dfrac{\sqrt{3}}{2} - i\dfrac{1}{2}$.

Thus c_0, c_1, c_2 are all the cube roots of $-i$, given in rectangular form.