Math 206 Complex Calculus
Quiz-3
Solutions

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1) Let $h(z)$ be an entire function with $h(0) = 0$, $h'(0) = 0$, $h''(0) = a \neq 0$ and $h'''(0) = b$. Setting $f(z) = 1/h(z)$, find $Res_{z=0} f(z)$.

Solution: Since h is entire it has a Taylor series at $z = 0$ which converges for all values of z. From the given data it follows that
$h(z) = \dfrac{a}{2}z^2 + \dfrac{b}{6}z^3 + \cdots = z^2(\dfrac{a}{2} + \dfrac{b}{6}z + \cdots)$,
and
$f(z) = \dfrac{1}{h(z)} = \dfrac{1}{z^2(\frac{a}{2} + \frac{b}{6}z + \cdots)} = \dfrac{1/(\frac{a}{2} + \frac{b}{6}z + \cdots)}{z^2}$.
Setting $\phi(z) = \dfrac{1}{\frac{a}{2} + \frac{b}{6}z + \cdots}$, we see that the required residue is equal to $\phi'(0)$ which is equal to $-\dfrac{2b}{3a^2}$.

If $h(z) = (e^z - 1)^2$, then $a = 2$, $b = 6$ and the residue is -1.
If $h(z) = \sin^2(z) + z^3$, then $a = 2$, $b = 6$ and the residue is -1.