1) Find all the fourth roots of (a) \( z_0 = -8 + 8\sqrt{3}i \), (b) \( z_0 = -8 - 8\sqrt{3}i \).

**Solution:** Let \( \alpha_0 = \text{Arg}(z_0) \), where \( 0 \leq \alpha_0 < 2\pi \). It is determined by plotting \( z_0 \) on the \( xy \)-plane. Now let \( \theta_k = \frac{\alpha_0 + 2k\pi}{4} \) for \( k = 0, 1, 2, 3 \). Note that in both cases \( |z_0| = 16 = 2^4 \), and \( c_k = 2(\cos \theta_k + i \sin \theta_k) \) for \( k = 0, 1, 2, 3 \) are all the required fourth roots of \( z_0 \).

**Solution a:** \( \alpha_0 = \frac{2\pi}{3} \).

\[ \theta_k = \left( \frac{2\pi}{3} + 2k\pi \right) \frac{1}{4}, \text{ so: } \theta_0 = \frac{\pi}{6}, \theta_1 = \frac{4\pi}{6} = \pi - \frac{\pi}{3}, \theta_2 = \frac{7\pi}{6} = \pi + \frac{\pi}{6}, \theta_3 = \frac{10\pi}{6} = 2\pi - \frac{\pi}{3}. \]

Now a straightforward calculation gives

\[ c_0 = \sqrt{3} + i, \ c_1 = -1 + i\sqrt{3}, \ c_2 = -\sqrt{3} - i, \ c_3 = 1 - i\sqrt{3}. \]

**Solution b:** \( \alpha_0 = \frac{4\pi}{3} \).

\[ \theta_k = \left( \frac{4\pi}{3} + 2k\pi \right) \frac{1}{4}, \text{ so: } \theta_0 = \frac{\pi}{3}, \theta_1 = \frac{5\pi}{6} = \pi - \frac{\pi}{3}, \theta_2 = \frac{8\pi}{6} = \pi + \frac{\pi}{3}, \theta_3 = \frac{11\pi}{6} = 2\pi - \frac{\pi}{6}. \]

Now a straightforward calculation gives

\[ c_0 = 1 + i\sqrt{3}, \ c_1 = -\sqrt{3} + i, \ c_2 = -1 - i\sqrt{3}, \ c_3 = \sqrt{3} - i. \]