Math 206 Complex Calculus
Quiz-4
Solutions

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1-a) Use Laplace transform techniques to solve the following differential equation:

$$f''(t) + f'(t) - 6f(t) = 50 \sin t, \quad \text{with } f(0) = -1, \; f'(0) = -7.$$

Solution 1-a:

$$\begin{aligned} \mathcal{L}(f(t)) &= F(s), \\ \mathcal{L}(f'(t)) &= sF(s) - f(0) = sF(s) + 1, \\ \mathcal{L}(f''(t)) &= s^2F(s) - sf(0) - f'(0) = s^2F(s) + s + 7, \\ \mathcal{L}(\sin(t)) &= \frac{1}{s^2+1}. \end{aligned}$$

Then the DE becomes;

$$\begin{aligned} (s^2 + s - 6)F(s) + (s + 8) &= \frac{50}{s^2+1}, \\ (s^2 + 2 - 6)F(s) &= \frac{-s^3 - 8s^2 - s + 42}{s^2+1}, \\ &= -\frac{(s^2 + 2 - 6)(s + 7)}{s^2+1}, \\ F(s) &= -\left(\frac{s}{s^2+1}\right) - 7\left(\frac{1}{s^2+1}\right). \end{aligned}$$

Recalling that

$$\begin{aligned} \mathcal{L}(\cos t) &= \frac{s}{s^2+1}, \quad \text{and} \\ \mathcal{L}(\sin t) &= \frac{1}{s^2+1}, \end{aligned}$$

we get:

$$f(t) = -\cos t - 7 \sin t.$$

1-b) Use Laplace transform techniques to solve the following differential equation:

$$f''(t) + 2f'(t) - 8f(t) = 85\cos t, \quad \text{with } f(0) = -9,\ f'(0) = 2.$$

Solution 1-b:

$$\begin{aligned}
\mathcal{L}(f(t)) &= F(s), \\
\mathcal{L}(f'(t)) &= sF(s) - f(0) = sF(s) + 9, \\
\mathcal{L}(f''(t)) &= s^2F(s) - sf(0) - f'(0) = s^2F(s) + 9s - 2, \\
\mathcal{L}(\cos(t)) &= \frac{s}{s^2+1}.
\end{aligned}$$

Then the DE becomes;

$$\begin{aligned}
(s^2 + 2s - 8)F(s) + (9s + 16) &= \frac{85s}{s^2+1}, \\
(s^2 + 2s - 8)F(s) &= \frac{-9s^3 - 16s^2 + 76s - 16}{s^2+1}, \\
&= -\frac{(s^2 + 2s - 8)(9s - 2)}{s^2+1}, \\
F(s) &= -9\left(\frac{s}{s^2+1}\right) + 2\left(\frac{1}{s^2+1}\right).
\end{aligned}$$

Recalling that

$$\begin{aligned}
\mathcal{L}(\cos t) &= \frac{s}{s^2+1}, \quad \text{and} \\
\mathcal{L}(\sin t) &= \frac{1}{s^2+1},
\end{aligned}$$

we get:

$$f(t) = -9\cos t + 2\sin t.$$
