1-a) Let $D = \{(x, y) \in \mathbb{R}^2 \mid x, y \geq 0 \}$. Find a bounded harmonic function $T(x, y)$ on $D$ such that $T(x, 0) = 0$ and $T(0, y) = 1$.

Solution 1-a: Map the region $D$ by $\log z = \ln r + i\theta = u + iv$ onto the region $E = \{(u, v) \in \mathbb{R}^2 \mid 0 \leq v \leq \pi/2 \}$. Consider the function $H(u, v) = 2v/\pi$ on $E$. This is harmonic on $E$ and satisfies the given boundary conditions. Let $T(x, y) = H(u(x, y), v(x, y))$. Putting in $v = \theta = \arctan(y/x)$ we get $T(x, y) = (2/\pi) \arctan(y/x)$ as the required bounded harmonic function.

Note: This is a simplified version of Exercise 4 on page 307.

1-b) Let $D = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1, \ x, y \geq 0 \}$. Find a harmonic function $T(x, y)$ on $D$ such that $T(x, 0) = 0$, $T(0, y) = 1$ and $\partial T/\partial \vec{n} = 0$ along the circular part of the boundary.

Solution 1-b: The function $\log z = \ln r + i\theta = u + iv$ maps $D$ onto the region $E = \{(u, v) \in \mathbb{R}^2 \mid u \leq 0, \ 0 \leq v \leq \pi/2 \}$. The function $H(u, v) = 2v/\pi$ is harmonic on $E$ and satisfies the given boundary conditions. Define $T(x, y) = H(u(x, y), v(x, y))$. Putting in $v = \theta = \arctan(y/x)$ we get $T(x, y) = (2/\pi) \arctan(y/x)$ as the required bounded harmonic function.

Note: This is a simplified version of Exercise 5 on page 308.