Math206 HW#8 Solutions

1)  
\[ \text{syms t} \]
\[ \text{laplace(t^3+2*t^2-t+1)} \]
\[ \text{ans =} \]
\[ \frac{6}{s^4}+\frac{4}{s^3}\frac{1}{s^2}+\frac{1}{s} \]
\[ \gg \text{laplace((t^2+2*t-1)*exp(2*t))} \]
\[ \text{ans =} \]
\[ \frac{2}{(s-2)^3}+\frac{2}{(s-2)^2}\frac{1}{(s-2)} \]

2) a) Taking the Laplace transform of both sides, we obtain;

\[ \frac{d^2}{dt^2} f(t) - \frac{d}{dt} f(t) - 2 f(t) = \delta(t - 1) \]
\[ (s^2F(s) - sf(0) - f'(0)) - (sF(s) - f(0)) - 2F(s) = e^{-s} \]
\[ (s^2 - s - 2)F(s) = e^{-s} \]
\[ F(s) = \frac{e^{-s}}{(s^2 - s - 2)} \]

b) Using MATLAB to inverse transform the result,

\[ \gg \text{syms s} \]
\[ \gg \text{ilaplace(exp(-s)/(s^2-s-2))} \]
\[ \text{ans =} \]
\[ -\frac{1}{3}\text{Heaviside}(t-1)\text{exp}(-t+1)+\frac{1}{3}\text{Heaviside}(t-1)\text{exp}(2*t-2) \]
\[ \gg \text{simple(ans)} \]
\[ \cdot\cdot\cdot \]
\[ \cdot\cdot\cdot \]
\[ \text{ans =} \]
\[ \frac{1}{3}\text{Heaviside}(t-1)\left(-\text{exp}(-t+1)+\text{exp}(2*t-2)\right) \]

We see that the result involves Heaviside functions, which is expected as the derivative of Heaviside functions yield the Delta-Dirac impulse function.
3) Since we are on the circle of radius $\frac{1}{2}$, $|z|=\frac{1}{2}$ in the region. Choose:

$$f(z) = 7z^2$$
$$g(z) = 4z^5 - 1$$

Taking absolute values;

$$|f(z)| = |7z^2| = 7 \cdot \left(\frac{1}{2}\right)^2 = 1.75$$
$$|g(z)| = |4z^5 - 1| \leq 4 \cdot \left(\frac{1}{2}\right)^5 + 1 = 1.125$$

→ Thus $|f(z)| > |g(z)|$ on the given circle and since $f(z)$ has 2 roots inside, so does $f(z) + g(z)$. Therefore inside the circle of radius $\frac{1}{2}$ there are two roots of the equation $4z^5 + 7z^2 - 1 = 0$.

Verification with MATLAB:

```matlab
>> roots([4 0 0 7 0 -1])
ans =
    0.5863 + 1.0774i
    0.5863 - 1.0774i
    -1.1609
    0.3725
    -0.3842
>> abs(ans)
ans =
    1.2266
    1.2266
    1.1609
    0.3725
    0.3842
```

→ Thus only 2 roots exist inside the circle of radius $\frac{1}{2}$.

4) Using MATLAB to evaluate the integral,

```matlab
>> syms x
>> int(sin(x)/x,0,inf)
ans =
    1/2*pi
```

This is same as the result seen on page 220 of the book.