

## Math 206 - Homework #3

Due February 28, 2006

1. Find a harmonic conjugate  $v(x, y)$  of  $u(x, y) = \cosh x \sin y$  such that  $v(0, 0) = 0$ .
2. Find a harmonic conjugate  $v(r, \theta)$  of  $u(r, \theta) = \ln r + \theta$  such that  $v(1, 0) = 0$ .
3. Find real  $a$  and  $b$  such that  $(1 + i)^{2006} = a + ib$ .
4. Find all values of  $z$  such that:
  - (a)  $e^z = -4$
  - (b)  $e^z = 2 + 2i$
  - (c)  $e^{(4z-2)} = -1$
5. Let a function  $f(z)$  be analytic in a domain  $D$ . Prove that  $f(z)$  must be constant in  $D$  if:
  - (a)  $f(z)$  is real-valued for all  $z$  in  $D$ .
  - (b)  $\overline{f(z)}$  is analytic in  $D$ .