MATH 206 HW5

1) Let C be the unit circle $z = e^{i\theta}\ (-\pi \le \theta \le \pi)$. First show that for any real constant a, $\int_C \frac{e^{az}}{z}\,dz = 2\pi i$. Then write the integral in terms of θ to derive the integration formula $\int_0^{\pi} e^{a\cos\theta}\cos(a\sin\theta)\,d\theta = \pi$.

2) Let C denote the positively oriented boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$. Evaluate each of the following integrals:

a) $\int_C \frac{\cos z}{z(z^2+8)}\,dz$ b) $\int_C \frac{\tan(z/2)}{(z-x_0)^2}\,dz\ (-2 < x_0 < 2)$ c) $\int_C \frac{\cosh z}{z^4}\,dz$

3) If C_0 denotes a positively oriented circle $|z - z_0| = R$, then show that

$$\int_{C_0} (z-z_0)^{n-1}\,dz = \begin{cases} 0, \text{ when } n = \pm 1, \pm 2, \pm 3, \pm 4, \textbf{h h h} \\ 2\pi i, \text{ when } n = 0 \end{cases}.$$

4) Let C be the circle $|z| = 3$, described in the positive sense. Show that if $g(w) = \int_C \frac{2z^2 - z - 2}{z - w}\,dz \quad (|w| \ne 3)$, then $g(2) = 8\pi i$. What is the value of $g(w)$ when $|w| > 3$?

5) Let C_R be the circle $|z| = R,\ (R > 1)$ described in the positive sense.

Show that $\left| \int_{C_R} \frac{Log\ z}{z^2}\,dz \right| < 2\pi \left(\frac{\pi + In\ R}{R} \right)$. Using this, show that the value of this integral tends to zero as $R \to \infty$ (R tends to infinity).