MATH 206 HW6

1) The PHASOR associated with a real-valued function $f(t)$ of time t is a complex number F, independent of t, such that $\mathrm{Re}\left[F\exp(st)\right]=f(t)$, where **s = σ+iω** is a complex number, called the complex frequency of $f(t)$.

(a) Determine the phasor representations and the corresponding complex frequencies for the functions:

(i) $f(t)=\cos(10t+\frac{\pi}{6})$ (ii) $f(t)=3e^{-t}\sin(5t)$ (iii) $f(t)=4e^{-3t}$

(b) Show that not all functions $f(t)$ have phasor representations.

2) Show that if f is analytic within and on a simple closed contour C and z_0 is not on C, then $\int_C \frac{f'(z)}{z-z_0}dz=\int_C \frac{f(z)}{\left(z-z_0\right)^2}dz$.

3) Show in two ways that the sequence $z_n=-2+i\frac{(-1)^n}{n^2}$ $(n=1,2,3,4,\ldots)$ converges to -2.

4) Show that when $0<|z|<4$, $\frac{1}{4z-z^2}=\frac{1}{4z}+\sum_{n=0}^{\infty}\frac{z^n}{4^{n+2}}$.

5) Let f be an entire function such that $\left|f(z)\right|\le A|z|^3$ for all z, where A is a fixed positive number. Show that $f(z)=a_1 z^3$, where a_1 is a complex constant.

<u>Hint :</u> Use Cauchy's inequality.

<u>Cauchy's inequality:</u> Let z_0 be a fixed complex number. If a function f is analytic within and on a circle $|z-z_0|=R$, taken in the positive sense and denoted by C and if $\left|f(z)\right|\le M_R$ on C, then

$$\left|f^{(n)}(z_0)\right|\le\frac{n!M_R}{R^n}\quad(n=1,2,3,\ldots)$$

$\left(f^{(n)}(z)\ \textit{means}\ n^{\text{th}}\ \text{derivative of f(z) with respect to z}\right)$