\( a) f(t) = \Re \left[ F e^{\sigma_t} \right] \\
= \Re \left[ F e^{\sigma t} + i \omega t \right] = \Re \left[ F e^{i \omega t} \right] \\
= e^{\sigma t} \left\{ \Re [F] \cos(\omega t) - \Im [F] \sin(\omega t) \right\}

\( i) f(t) = \cos \left( 2\omega t + \frac{\pi}{2} \right) = \Re \left[ e^{i \frac{\pi}{2}} e^{2i\omega t} \right] \\
\sigma = 0 \quad F = e^{i \frac{\pi}{2}} \rightarrow \text{phasor} \\
\omega = 10 \quad s = i \cdot 10 \rightarrow \text{complex frequency}

\( ii) f(t) = 2e^{-t} \sin(3t) = e^{-t} \Re \left[ 2 e^{-i \frac{\pi}{2}} e^{3i\omega t} \right] \\
\sigma = -1 \quad F = 2e^{-i \frac{\pi}{2}} \rightarrow \text{phasor} \\
\omega = 5 \quad s = -1 + 5i \rightarrow \text{complex frequency}

\( iii) f(t) = 4e^{-2t} = e^{-2t} \Re \left[ 4 e^{i \omega t} \right] \\
\sigma = -2 \quad F = 4 \rightarrow \text{phasor} \\
\omega = 0 \quad s = -2 \rightarrow \text{complex frequency}
b) If \( f(t) \) has more than one frequency component, then it won't have a phasor representation. For example, if \( f(t) = (\cos t) + (\cos 2t) \) then \( f(t) \) can't be represented by \( \text{Re} \left[ F \cdot e^{jt} \right] \) where \( F \) is independent of \( t \). Such functions have more than one complex frequency and as a result they don't have a phasor \( F \) which is independent of \( t \). For \( f(t) = (\cos t) + (\cos 2t) \), 

\[
f(t) = (\cos t) + (\cos 2t) = \text{Re} \left[ e^{jt} + e^{j2t} \right]
\]
e\( jt \) and \( e^{j2t} \) can't be represented by \( F = \text{Re} \left[ e^{j\omega t} \right] \) where \( F \) is independent of \( t \).

\[
e^{jt} + e^{j2t} = e^{jt} \left[ 1 + e^{j\pi t} \right]
\]

We don't have a \( F \) independent of \( t \).
If \( f(z) \) and \( f'(z) \) are analytic within and on \( C \), \( z_0 \) is not on \( C \). There are two cases:

1. \( z_0 \) is in \( C \)
2. \( z_0 \) is outside \( C \)

For case 1. If \( \frac{f'(z)}{z-z_0} \) is analytic on \( C \),

\[
\oint_C \frac{f'(z)}{z-z_0} \, dz = z_0 \cdot \pi \cdot i \cdot f'(z_0)
\]

Cauchy Integral Formula

for \( z_0 \) is outside \( C \),

\[
\frac{f''(z)}{z-z_0} \quad \text{and} \quad \frac{f'(z)}{(z-z_0)^2}
\]

are analytic within and on \( C \).

Since \( C \) is a simple closed contour, we have:

\[
\oint_C \frac{f''(z)}{z-z_0} \, dz = \oint_C \frac{f'(z)}{(z-z_0)^2} \, dz = 0
\]

So, \( \oint_C \frac{f''(z)}{z-z_0} \, dz \) and \( \oint_C \frac{f'(z)}{(z-z_0)^2} \, dz \)

are always equal under given conditions.
\[ z_n = (z - i) + i \cdot \frac{(-i)\sqrt{n}}{n^2} = x_n + i \cdot y_n \]

\[ x_n = -z \quad y_n = \frac{(-i)\sqrt{n}}{n^2} \]

\[ \lim_{n \to \infty} z_n = \lim_{n \to \infty} x_n + i \cdot \lim_{n \to \infty} y_n \]

\[ \lim_{n \to \infty} x_n = -z \quad \lim_{n \to \infty} y_n = \lim_{n \to \infty} \frac{-i\sqrt{n}}{n^2} = 0 \]

So, \( \lim_{n \to \infty} z_n = -z + i \cdot 0 = -z \)

We will also show that \( z_n \) converges to \( -z \) by using the definition of limit.

If \( z_n \) converges to \( (-2) \), then for every positive number \( \varepsilon \), there should exist a positive integer \( N \) such that

\[ |z_n - (-2)| < \varepsilon \quad \text{whenever} \quad n > N_0. \]

\[ |z_n - (-2)| = \left| i \cdot \frac{-i\sqrt{n}}{n^2} \right| = \frac{1}{\sqrt{n}} \]

\[ |z_n - (-2)| < \varepsilon \quad \Rightarrow \quad \frac{1}{\sqrt{n}} < \varepsilon \quad \Rightarrow \quad n > \frac{1}{\varepsilon^2} \]

So if we choose \( N_0 \) as the biggest integer which is smaller than or equal to \( \frac{1}{\varepsilon^2} \), the desired condition is satisfied.

This means that we can find a \( N_0 \) for every positive \( \varepsilon \) value. This concludes the proof.

\[ \lim_{n \to \infty} z = -2. \]
\[
\frac{1}{4z - 2z^2} = \frac{1}{4z} \cdot \frac{1}{1 - \frac{z}{4}}
\]

\[
\frac{1}{z - \frac{z}{4}} = \sum_{n=0}^{\infty} \left( \frac{z}{4} \right)^n \quad \text{when} \quad \left| \frac{z}{4} \right| < 1.
\]

So when \( |z| < 4 \),

\[
\frac{1}{4z - 2z^2} = \frac{1}{4z} \cdot \frac{1}{1 - \frac{z}{4}}
\]

is singular at \( z = 0 \).

So when \( 0 < |z| < 4 \),

\[
\frac{1}{4z - 2z^2} = \frac{1}{4z} \cdot \sum_{n=0}^{\infty} \left( \frac{z}{4} \right)^n = \sum_{n=0}^{\infty} \frac{z^n}{4^{n+1}}
\]

\[
= \sum_{n=-1}^{\infty} \frac{z^n}{4^{n+2}} = \frac{1}{4z} + \sum_{n=0}^{\infty} \frac{z^n}{4^{n+2}}
\]
5) Since \( f(z) \) is entire, it satisfies Cauchy's inequality for every \( z \) and \( R \) value. We should find \( M_R \) on \( C: z = z_0 + R \cdot e^{i\theta} \), \(-\pi < \theta < \pi\)

\[
\left| f(z) \right| \leq A \cdot |z|^2 \leq A \cdot (|z_0| + R)^2
\]

\[
M_R
\]

So for every \( z \) and \( R \) value

\[
\left| f^{(n)}(z_0) \right| \leq \frac{n! \cdot A \cdot (|z_0| + R)^2}{R^n}
\]

for \( n = 1, 2, 3, \ldots \)

If we let \( R \to \infty \), \( \left| f^{(n)}(z_0) \right| \leq 0 \) for every \( z \) value.

This means that \( f^{(n)}(z_0) = 0 \) for every \( z \) value.

So \( f^{(n)}(z) = 0 \), \( \to f(z) = c \to \text{constant} \)

\[
f^{(2)}(z) = c \cdot z + d \to \text{constant}
\]
\[ f^{(21)}(z_0) \leq \frac{2A \left( 12z_0 + R \right)^2}{R^2} \]

We also know \( f^{(21)}(z) = c\cdot z + d \).

If we let \( z_0 = 0 \), we get:

\[ \left| f^{(21)}(0) \right| = |d| \leq \frac{2A \cdot R^2}{R^2} \]

for every possible \( R \) value.

If we let \( R \to 0 \), we get:

\[ \lim_{R \to 0} |d| \leq 0 \]

\[ d = 0 \]

\[ f^{(21)}(z) = c \cdot z \]

\[ f^{(31)}(z_0) \leq \frac{A \left( 12z_0 + R \right)^2}{R} \]

If we let \( z_0 = 0 \), we get:

\[ \left| f^{(31)}(0) \right| = |e| \leq A \cdot R^2 \]

for every possible \( R \) value.

If we let \( R \to 0 \), we get:

\[ \lim_{R \to 0} |e| \leq 0 \]

\[ e = 0 \]

\[ f^{(41)}(z) = \frac{c}{2} \cdot z^2 \]

\[ f^{(42)}(z) = \frac{c}{6} \cdot z^3 + b \rightarrow \text{constant} \]

\[ f(0) = b \quad \text{We know} \quad |f(z)| \leq A \cdot |z|^2 \]
\[ f(x) = \frac{1}{3}x^3 \]

So

\[ f(0) = \frac{1}{3}a \]

Where \( a \) and \( b \) are constants.