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Time: 9:30-11:30
Instructor: Ali Sinan Sertöz

Math 300
A Concise History of Mathematics
Final Exam
Sample Answer Paper

1-a) Defend the thesis that military funding is necessary for the advancement of mathematics.

Military funding points in the direction of useful research and promotes work. Even the construction of the pyramids in Ancient Egypt had a military desire for superiority and this in turn resulted in recognition of mathematics as a major force in running a state. Astronomy and associated mathematics were supported by naval forces for a long time to develop tools and means of navigating the open seas. Conic sections were studied vehemently long after the Greeks because of their applicability to artillery. Even when military funding is directed to some big engineering problems it helps indirectly to the development of mathematics through provoking research in related areas of mathematics. Hence we can safely argue that military funding is indispensable for the advancement of mathematics.

1-b) Defend the thesis that military funding is detrimental for the advancement of mathematics.

The direction military funding encourages research is unnatural from a purely mathematical point of view. Practicability is not the direction pure mathematics should necessarily develop. We can argue that the ruler and compass type of research was popular for a long time because it is easy to employ this in the army and hence military did not encourage for a long time research in understanding geometry using more complicated tools, and this naturally hindered the free development of mathematics. Moreover the military secrecy prevented the dissemination of knowledge freely across borders for centuries and mathematicians in different sides of the cold war, for example, had to discover the same results separately and thus wasted time. Without dependence on military funding mathematics will develop much more naturally and freely.

2-a) Defend the thesis that the obsession of the Ancient Greeks with geometry hindered the proper development of mathematics.

The Ancient Greeks ignored number theory. They tried to explain every mathematical concept through geometry and this in turn restricted their vocabulary of mathematical ideas. They did not introduce numerical measurements into geometry and thus delayed the development of analysis. They were so obsessed with the the idea of perfection, which incidentally finds its best manifestation in geometry, that they totally ignored the idea of chance and thus the theory of probability was not studies until the seventeenth century. Their obsession with geometry can be attributed to their devotion to the illusion of perfection in nature.

2-b) Defend the thesis that the obsession of the Ancient Greeks with geometry accelerated the proper development of mathematics.

Their devotion to geometry gave a solid base for mathematics to develop. The other branches of mathematics had a challenge before them; to match the degree of perfection to which the Ancient Greeks carried geometry. Their attempts to give proofs and axiomatic approaches helped mathematics to emancipate itself from the ad hoc attempts of the curious and made it advance to the heights of academic endeavor which it enjoys ever since. Their introduction of ruler and compass problems for example stimulated mathematical research during the next two millennia.

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3) Since there are efficient ways of getting good approximations to the roots of cubics, there is no point in having an algebraic formula giving the exact roots. Cardano's work was therefore a waste of time. Comment on this.

I will argue that this remark fails to recognize the true nature of science. Mathematics in particular is about understanding the underlying concepts of any problem and not necessarily about giving everyday solutions to problems. In this vein we argue that mathematicians in general, and Cardano in particular, were right in attempting to find exactly the true description of the roots. They were not interested in finding the roots; they were interested in *understanding* the roots. Moreover their abstract work can still be praised from a practical point of view too. For example the infinite series known as the Harmonic Series, $\sum_{n=1}^{\infty} 1/n$, diverges without any empirical evidence whatsoever. A naive approach can give some numerical approximations for the sum based on the values suggested by the first few thousand terms. A theoretical understanding of the underlying concepts prevents such embarrassments. Thus Cardano's search for the exact roots contributed to mathematics par excellence.

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4) The author of the textbook at some point complains that "Because they must 'publish or perish', second rate mathematicians fill the journals with useless abstractions." Comment on this.

First of all this sentence can easily be translated into "I don't understand most this work." Failure to appreciate what you don't understand is precisely what education is aiming to eliminate. From a mathematical point of view it is not possible to classify with some degree of accuracy any work done as useless. You allow mathematics to develop in its natural path and never know when one particular piece of work is going to be significant in the future. It is possible to make some educated guesses but they are after all only guesses. The works of Riemann and Hilbert for example proved to be significant in some physical models of the universe later on. Imagine where we would be today if our author had the authority to prevent Riemann to publish his *useless abstractions* back in the nineteenth century.

5-a) Argue in favor of the view that mathematical objects exist independently of the human mind.

Many independent mathematical discoveries later glue in to give a global picture of a grand scheme which was not anticipated by any of the individual contributors. This is in general the most striking evidence which influences mathematicians to come to the inevitable conclusion that what they are working on already exists *somewhere*. For example now we know that the non-Euclidean geometry explains our universe much more precisely than Euclidean geometry. We can therefore argue that the non-Euclidean geometry existed in space long before it was discovered by the mortals who lived in that space.

5-b) Argue against the view that mathematical objects exist independently of the human mind.

Mathematical objects do not exist; they can only be conceived. For example you can see the 'green' and the 'apple' in "one green apple" but you can only conceive the 'one'. Starting from such elementary level it is possible to argue that none of the objects of mathematics exists independently of the human mind, though mathematical objects when conceived correctly manifest their effects on everyday life. Mathematical objects help us to understand the shortest distance between two cities on this globe but the actual path taken by a plane is not exactly what the mathematician has conceived. The route of the plane exists but it is not a mathematical object and the mathematical object which is the shortest distance does not exist.
