Due on November 8, 2006, Wednesday, Class time. No late submissions!

MATH 302 Homework 2

1: Let $H = \{ z \in \mathbb{C} \mid Im \ z \geq 0 \}$ and $f : H \rightarrow \mathbb{C}$ be a nonconstant analytic function with $\sup_{z \in H} |f(z)| = 1$. Construct such a function $f$ with
\begin{enumerate}[(i)]  
\item $|f(z_0)| = 1$ for some $z_0 \in H$.
\item $|f(z)| < 1$ for all $z \in H$.
\end{enumerate}

2: Let $D = \{ z \in \mathbb{C} \mid Re \ z \geq 0 \}$. Construct a nonconstant analytic function $f : D \rightarrow \mathbb{C}$ with $|f(z)| \leq 1$ on $D$ such that for every $\epsilon > 0$ there is a corresponding $A_\epsilon \in \mathbb{R}$ with $|f(z)| \leq A_\epsilon e^{\epsilon|z|}$ for all $z \in D$.

3: Find a counterexample to Corollary 16.6 on page 202 when $D$ is a proper subset of $\mathbb{C}$ but is not compact.

4: Consider the function $g(z)$ constructed in the proof of Theorem 16.8 on page 205. Show that
\begin{enumerate}[(i)]  
\item $g$ is continuous in the unit disk.
\item $g$ is analytic in the unit disk.
\end{enumerate}

5: Find a $C$-harmonic function $u(x, y)$ on the unit disk $D$ with $u(x, y) = ax^2 + bxy + cy^2 + dx + ey + f$ on $\partial D$, where the $a, b, \ldots, f$ are arbitrary real constants.

Show your work in detail. Justify all your claims. No correct answer is accepted unless you prove that it is correct!