Due on November 8, 2006, Wednesday, Class time. No late submissions!

MATH 302 Homework 2

1: Let $H = \{z \in \mathbb{C} \mid Im\ z \geq 0\ \}$ and $f : H \longrightarrow \mathbb{C}$ be a nonconstant analytic function with $\sup_{z \in H} |f(z)| = 1$. Construct such a function f with
(i) $|f(z_0)| = 1$ for some $z_0 \in H$.
(ii) $|f(z)| < 1$ for all $z \in H$.

2: Let $D = \{z \in \mathbb{C} \mid Re\ z \geq 0\ \}$. Construct a nonconstant analytic function $f : D \longrightarrow \mathbb{C}$ with $|f(z)| \leq 1$ on D such that for every $\epsilon > 0$ there is a corresponding $A_\epsilon \in \mathbb{R}$ with $|f(z)| \leq A_\epsilon e^{\epsilon|z|}$ for all $z \in D$.

3: Find a counterexample to Corollary 16.6 on page 202 when D is a proper subset of $\mathbb{C}$ but is not compact.

4: Consider the function $g(z)$ constructed in the proof of Theorem 16.8 on page 205. Show that
(i) g is continuous in the unit disk.
(ii) g is analytic in the unit disk.

5: Find a C-harmonic function $u(x, y)$ on the unit disk D with $u(x, y) = ax^2 + bxy + cy^2 + dx + ey + f$ on ∂D, where the $a, b, \ldots, f$ are arbitrary real constants.

Show your work in detail. Justify all your claims. No correct answer is accepted unless you prove that it is correct!