Due on December 11, 2006, Monday, Class time. No late submissions!

MATH 302 Homework 3

1: Find an integral expression for $\Gamma'(z)$ for $\text{Re } z > 0$. Justify your steps.

2: Prove that $\Gamma(z) = \lim_{n \to \infty} \int_0^\infty t^{z-1} \left(1 - \frac{t}{n}\right)^n dt$, $\text{Re } z > 0$.

3: Find the radius of convergence for $f(z) = \sum_{n=0}^\infty z^n$ and show that its circle of convergence is a natural boundary.

4: Show that $\Gamma(z + 1) = z\Gamma(z)$ for all $z \in \mathbb{C}$ except for $z = -n$ where $n \in \mathbb{N}$.

5: Show that $\sum_{p: \text{ prime}} \frac{1}{p}$ diverges.