Due on December 11, 2006, Monday, Class time. No late submissions!

MATH 302 Homework 3

1: Find an integral expression for $\Gamma'(z)$ for $Re \ z > 0$. Justify your steps.

2: Prove that $\Gamma(z) = \lim\limits_{n \to \infty} \int_0^\infty t^{z-1} \left(1 - \dfrac{t}{n}\right)^n \, dt$, $Re \ z > 0$.

3: Find the radius of convergence for $f(z) = \sum\limits_{n=0}^{\infty} z^{n!}$ and show that its circle of convergence is a natural boundary.

4: Show that $\Gamma(z+1) = z\Gamma(z)$ for all $z \in \mathbb{C}$ except for $z = -n$ where $n \in \mathbb{N}$.

5: Show that $\sum\limits_{p: \ \text{prime}} \dfrac{1}{p}$ diverges.