Math 302 Complex Calculus II – Homework

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Please do not write anything inside the above boxes!

Check that there are 2 questions on your booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.

Q-3) Classify all the automorphisms of the first quadrant

Solution:
Q-4) This exercise aims to complete the proof of a theorem we did in class.

Fix $\alpha \in \mathbb{C}$ with $|\alpha| < 1$. Define

$$h(z) = \left( \frac{z - i}{z + i} \right)^{-1} \circ \left( \frac{z - \alpha}{1 - \bar{\alpha} z} \right) \circ \left( \frac{z - i}{z + i} \right).$$

Show that

$$h(z) = \frac{az + b}{cz + d} \quad \text{with} \quad a, b, c, d \in \mathbb{R} \quad \text{and} \quad ad - bc > 0.$$

Solution: