Due Date: June 27, 2011 Monday NAME:..

Ali Sinan Sertöz STUDENT NO:..

Math 302 Complex Analysis II – Homework 4 – Solutions

1	2	TOTAL
10	10	20

Please do not write anything inside the above boxes!

Check that there are 2 questions on your booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.

NAME: STUDENT NO:

Q-1) Let R be the complex plane with the non-positive real axis taken out. Find explicitly a conformal mapping f of R onto the unit disc U such that $f(1) = 0$ and $f'(1) > 0$.

Solution:

Take the principal branch of $\log$ function and define a square root function such that $\sqrt{1} = 1$.

First note that $z \mapsto \sqrt{z}$ maps R to all z with strictly positive real parts and such points have distance strictly larger than 1 from the point -1. So the map $g(z) = \dfrac{1}{\sqrt{z}+1}$ sends R conformally into U. Note that $g(1) = 1/2$.

Now using Theorem 13.15 from the book, we can consider the map

$$f(z) = \frac{2g(z) - 1}{g(z) - 2}, \; x \in R.$$

Check that $f(1) = 0$ and $f'(1) = 1/6 > 0$.

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Q-2) Let S be the Archimedean spiral given parametrically as

$$x(t) = t \cos t, \ y(t) = t \sin t, \quad t \in [0, \infty).$$

Let R be the complement of S in $\mathbb{C}$.

Can you define a branch of log function on R? If *yes*, construct this branch. If *no*, explain why.

Is R still conformal to the open unit disc?

Solution:

First of all, since R is a simply connected, proper open subset of $\mathbb{C}$, it is conformally isomorphic to the unit disc by the Riemann mapping theorem.

To construct a $\log$ function on R, which is essential in proving the existence of such an isomorphism, fix a point w_0 in the complement of R. Also fix a point z_0 in R.

For any point z in R, let C_z be a path from z_0 to z lying totally in R. Define

$$\log z := \int_{C_z} \frac{dz}{z - w_0}.$$

Since R is simply connected, the integral is independent of which path chosen as long as the path lies in R.

This then defines an explicit branch of the logarithm function.