Due Date: June 30, 2011 Thursday NAME:..

Ali Sinan Sertöz STUDENT NO:..

Math 302 Complex Analysis II – Homework 5

1	2	TOTAL
10	10	20

Please do not write anything inside the above boxes!

Check that there are 2 questions on your booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.

NAME: STUDENT NO:

Q-1) Show that $e^{e^z} \ll 1$ throughout the boundary of the region

$$D = \{x + iy \mid -\frac{\pi}{2} < y < \frac{\pi}{2}\}.$$

Show that it is the "smallest" such analytic function.

This is Exercise 3 of your textbook at the end of the chapter on Maximum-Modulus Theorems. (page 199 on 2nd edition, page 223 on 3rd edition.)

Explain what the problem is asking for. Give your solution in detail. There is a clue given at the back of the book which does not count as a solution!

Solution:

NAME: STUDENT NO:

Q-2) We proved the following theorem: If f is a non-constant entire function, there exists a curve along which f approaches infinity.

(i) Explain why Liouville's theorem does not immediately guarantee the existence of such a curve. Does there exist a non-constant entire function f and a path γ such that f does not go to infinity along γ even though $\gamma(t) \to \infty$ as $t \to \infty$? If *yes*, give an example, if *no*, explain why.

(ii) In the proof of the above theorem we explicitly used the polygonally connectedness of certain open sets which were connected components of some crucially defined sets. Prove that these sets were actually polygonally connected. (i.e., using the notation of the book, show that each S_k is polygonally connected.)

Solution: