Due Date: November 29, 2013 Friday NAME:..

Ali Sinan Sertöz STUDENT NO:..

Math 302 Complex Analysis II – Homework 2

1	2	3	4	TOTAL
10	10	10	10	40

Please do not write anything inside the above boxes!

Check that there are 3 questions on your booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.

NAME: STUDENT NO:

Q-1) Let L be a line in the complex plane and let T be a Mobius transformation sending L again to a line. Classify all such T.

Solution:

NAME: STUDENT NO:

Q-2) Let T be the Mobius transformation with $T(i\sqrt{3}) = \infty$, $T(-i\sqrt{3}) = 0$, and $T(0) = -1$. Find the image under T of the region R which is the intersection of the discs $|z+1| \leq 2$ and $|z-1| \leq 2$.

Solution:

NAME: STUDENT NO:

Q-3) Let R be a connected, simply connected, non-empty, proper open subset of $\mathbb{C}$. Fix a point $z_0 \in R$. Let $\mathcal{F}$ be the set of all analytic injective functions f from R into the unit disc U with $f(z_0) = 0$. Assume that $\mathcal{F}$ is not empty. Take an $f \in \mathcal{F}$. Show that if f is not surjective, then there is some $g \in \mathcal{F}$ such that $|g'(z_0)| > |f'(z_0)|$.

Solution:

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Q-4) Suppose f is an entire function and there is some $R > 0$ and $z_0 \in \mathbb{C}$ such that the open ball $B_R(z_0)$ with radius R centered at z_0 is not in the range of f, i.e. $B_R(z_0) \cap f(\mathbb{C}) = \emptyset$. Show that f is constant. *(In fact if the complement of $f(\mathbb{C})$ contains at least two distinct points, let alone a whole disc, then f is constant. That is Picard's theorem but you must demonstrate a proof for this homework problem only using Liouville's theorem.)*

Solution: