Due Date: November 29, 2013 Friday

NAME:............................................................

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STUDENT NO:............................................................

Math 302 Complex Analysis II – Homework 2

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*Please do not write anything inside the above boxes!*

Check that there are 3 questions on your booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.
Q-1) Let $L$ be a line in the complex plane and let $T$ be a Mobius transformation sending $L$ again to a line. Classify all such $T$.

Solution:
Q-2) Let $T$ be the Mobius transformation with $T(i\sqrt{3}) = \infty$, $T(-i\sqrt{3}) = 0$, and $T(0) = -1$. Find the image under $T$ of the region $R$ which is the intersection of the discs $|z+1| \leq 2$ and $|z-1| \leq 2$.

Solution:
Q-3) Let $R$ be a connected, simply connected, non-empty, proper open subset of $\mathbb{C}$. Fix a point $z_0 \in R$. Let $\mathcal{F}$ be the set of all analytic injective functions $f$ from $R$ into the unit disc $U$ with $f(z_0) = 0$. Assume that $\mathcal{F}$ is not empty. Take an $f \in \mathcal{F}$. Show that if $f$ is not surjective, then there is some $g \in \mathcal{F}$ such that $|g'(z_0)| > |f'(z_0)|$.

Solution:
Q-4) Suppose $f$ is an entire function and there is some $R > 0$ and $z_0 \in \mathbb{C}$ such that the open ball $B_R(z_0)$ with radius $R$ centered at $z_0$ is not in the range of $f$, i.e. $B_R(z_0) \cap f(\mathbb{C}) = \emptyset$. Show that $f$ is constant. (In fact if the complement of $f(\mathbb{C})$ contains at least two distinct points, let alone a whole disc, then $f$ is constant. That is Picard’s theorem but you must demonstrate a proof for this homework problem only using Liouville’s theorem.)

Solution: