Due Date: December 13, 2013 Friday NAME:..

Ali Sinan Sertöz STUDENT NO:..

Math 302 Complex Analysis II – Homework 3

1	2	3	4	TOTAL
10	10	10	10	40

Please do not write anything inside the above boxes!

Check that there are 3 questions on your booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.

NAME: STUDENT NO:

Q-1) Let f be an entire function and suppose that there is a positive integer n and positive real numbers A and R such that for all $z \in \mathbb{C}$ with $|z| \geq R$, we have $|f(z)| \leq A|z|^n$. Use Cauchy Integral Formula to show that f is a polynomial of degree at most n.

Solution:

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Q-2) Let $f = u + iv$ be an entire function. Theorem 16.10 says that if $|u(z)| \leq A|z|^n$ for all sufficiently large z, and for some constant $A > 0$ and for some non-negative integer n, then f is a polynomial of degree at most n. The proof uses Theorem 16.9 which says that if f is C-analytic in $D(0, R)$, for some $R > 0$, then

$$f(z) = \frac{1}{2\pi} \int_0^{2\pi} u(Re^{i\theta}) \frac{Re^{i\theta} + z}{Re^{i\theta} - z} \, d\theta + iv(0).$$

Assuming that differentiation with respect to z can be carried inside the integral sign for all orders, give an alternate proof of Theorem 16.10 by showing that $f^{(n+1)}(z_0) = 0$ for all $z_0 \in \mathbb{C}$.

Solution:

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Q-3) Let $u(x, y)$ and $v(x, y)$ be harmonic functions on a region D in $\mathbb{C}$. Find conditions on u and v such that uv is harmonic on D. Show that these conditions hold if $u + iv$ is analytic on D. Show however that when uv is harmonic, it does not necessarily imply that $u + iv$ is analytic on D.

Solution:

NAME: STUDENT NO:

Q-4) Find the Weierstrass product form of the entire function $\sinh z$.

Solution: