Due Date: 15 February, Monday 2016
Time: Class time
Instructor: Ali Sinan Sertöz

NAME:..

STUDENT NO:.......................................

Math 302 Complex Analysis II – Homework 1 – Solutions

1	2	3	4	5	TOTAL
50	50	0	0	0	100

Please do not write anything inside the above boxes!

Check that there are **2** questions on your exam booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.
Submit your solutions on this booklet only. Use extra pages if necessary.

Rules for Homework Assignments

(1) You may discuss the problems only with your classmates or with me. In particular you may not ask your assigned questions or any related question to online forums.

(2) You may use any written source be it printed or online. Google search is perfectly acceptable.

(3) It is absolutely mandatory that you write your answers alone. Any similarity with your written words and any other solution or any other source that I happen to know is a direct violation of honesty.

(4) You must obey the usual rules of attribution: all sources you use must be explicitly cited in such a manner that the source is easily retrieved with your citation. This includes any ideas you borrowed from your friends. (It is a good thing to borrow ideas from friends but it is a bad thing not to acknowledge their contribution!)

(5) Even if you find a solution online, you must rewrite it in your own narration, fill in the blanks if any, making sure that you exhibit your total understanding of the ideas involved.

Affidavit of compliance with the above rules: I affirm that I have complied with the above rules in preparing this submitted work.

Please sign here:

NAME: STUDENT NO:

Q-1) Consider the infinite sum $\sum_{n=1}^{\infty} \frac{1}{n^4+1} = \frac{1}{2} + \frac{1}{17} + \frac{1}{82} + \frac{1}{257} + \cdots$.

Follow the following steps to find the numerical value of this sum.

1. Find the poles of $\frac{\pi \cot \pi z}{z^4+1}$.
2. Calculate the residue of $\frac{\pi \cot \pi z}{z^4+1}$ at each pole.
3. Using WolframAlpha or some other software, find the numerical sum of all residues.
4. Using the above information find the value of the infinite sum $\sum_{n=1}^{\infty} \frac{1}{n^4+1}$.
5. Using WolframAlpha or some other software, find the value of the infinite sum $\sum_{n=1}^{\infty} \frac{1}{n^4+1}$ directly.

Solution:

The poles are the zeros of z^4+1. These roots are $z_k = \exp((2k+1)\pi/4)$ for $k = 0, 1, 2, 3..$ These are simple zeros and the numerator, $\pi \cot \pi z$ does not vanish at these points. Therefore these are simple poles of $\frac{\pi \cot \pi z}{z^4+1}$. Hence the residues can be calculated easily as follows.

$$R_k = \mathrm{Res}(\frac{\pi \cot \pi z}{z^4+1}, z = z_k) = \left(\left. \frac{\pi \cot \pi z}{4z^3} \right|_{z=z_k} \right).$$

Calculating these we find
$R_1 = -0.5392387900 + 0.5642639625\, i,\ R_2 = -0.5392387900 - 0.5642639625\, i,$
$R_3 = -0.5392387900 + 0.5642639625\, i,\ R_4 = -0.5392387900 - 0.5642639625\, i.$

Summing these up we get

$$R = R_1 + R_2 + R_4 + R_4 = -2.156955159.$$

The general theory says that

$$-R = \sum_{n=-\infty}^{\infty} \frac{1}{n^4+1} = 1 + 2\sum_{n=1}^{\infty} \frac{1}{n^4+1}.$$

We then find from general theory that

$$\sum_{n=1}^{\infty} \frac{1}{n^4+1} = \frac{-R-1}{2} = .5784775800.$$

This is compatible with the WolframAlpha output

$$\sum_{n=1}^{\infty} \frac{1}{n^4+1} = 0.57847757966713683831802219324571923504667221 7327....$$

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Q-2) Consider the infinite sum $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4+1} = \frac{1}{2} - \frac{1}{17} + \frac{1}{82} - \frac{1}{257} + \cdots$.

Follow the following steps to find the numerical value of this sum.

1. Find the poles of $\frac{\pi \csc \pi z}{z^4+1}$.
2. Calculate the residue of $\frac{\pi \csc \pi z}{z^4+1}$ at each pole.
3. Using WolframAlpha or some other software, find the numerical sum of all residues.
4. Using the above information find the value of the infinite sum $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4+1}$.
5. Using WolframAlpha or some other software, find the value of the infinite sum $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4+1}$ directly.

Solution:

The poles are the zeros of z^4+1. These roots are $z_k = \exp((2k+1)\pi/4)$ for $k = 0, 1, 2, 3..$ These are simple zeros and the numerator, $\pi \csc \pi z$ does not vanish at these points. Therefore these are simple poles of $\frac{\pi \csc \pi z}{z^4+1}$. Hence the residues can be calculated easily as follows.

$$R_k = \text{Res}(\frac{\pi \csc \pi z}{z^4+1}, z = z_k) = \left(\frac{\pi \csc \pi z}{4z^3}\bigg|_{z=z_k} \right).$$

Calculating these we find
$R_1 = -0.02471407440 - 0.1680063460\,i,\ R_2 = -0.02471407440 + 0.1680063460\,i,$
$R_3 = -0.02471407440 - 0.1680063460\,i,\ R_4 = -0.02471407440 + 0.1680063460\,i.$

Summing these up we get

$$R = R_1 + R_2 + R_4 + R_4 = -0.09885629760.$$

The general theory says that

$$-R = \sum_{n=-\infty}^{\infty} \frac{(-1)^n}{n^4+1} = 1 - 2\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4+1}.$$

We then find from general theory that

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4+1} = \frac{R+1}{2} = 0.4505718512.$$

This is compatible with the WolframAlpha output

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4+1} = 0.45057185128012682077070640194123388496782648168 0....$$