Due Date: 10 February 2015, Tuesday Time: Class time Instructor: Ali Sinan Sertöz



NAME:....

STUDENT NO:

# Math 430 / Math 505 Introduction to Complex Geometry – Homework 1

1	2	3	4	5	TOTAL
20	40	40	0	0	100

Please do not write anything inside the above boxes!

Check that there are **3** questions on your exam booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit. **Submit your solutions on this booklet only. Use extra pages if necessary.** 

# **Rules for Homework and Take-Home Exams**

- (1) You may discuss the problems only with your classmates or with me. In particular you may not ask your assigned questions or any related question to online forums.
- (2) You may use any written source be it printed or online. Google search is perfectly acceptable.
- (3) It is absolutely mandatory that you write your answers alone. Any similarity with your written words with any other solution or any other source that I happen to know is a direct violation of honesty.
- (4) You must obey the usual rues of attribution: all sources you use must be explicitly cited in such a manner that the source is easily retrieved with your citation. This includes any ideas you borrowed from your friends.
- (5) Even if you find a solution online, you must rewrite it in your own narration, fill in the blanks if any, making sure that you exhibit your total understanding of the ideas involved.
- (6) If I sense any hint of violation of any of the above rules in your paper I will either assume that you did not answer that question or that your contribution is only on the level of a scribe in which case I will assign a small fraction of the grade for that question.

Affidavit of compliance with the above rules: I affirm that I have complied with the above rules in preparing this submitted work. Moreover I declare that every solution I wrote reflects my true understanding of the problem and any sources used, ideas from friends or other sources included, are explicitly cited without exception. I am aware that violation of the above codes results not only in losing some meaningless grades but also of respect and dignity.

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**Q-1**) Let  $\epsilon = (\epsilon_1, \epsilon_2)$  with  $\epsilon_1, \epsilon_2 > 0$ . Let  $B_{\epsilon}(0)$  be the polydisc around the origin defined by

$$B_{\epsilon}(0) = \{(z_1, z_2) \in \mathbb{C}^2 \mid |z_1| < \epsilon_1 \text{ and } |z_2| < \epsilon_2\}.$$

- (a) Describe the set  $\partial B_{\epsilon}(0)$ , the boundary of the polydisc  $B_{\epsilon}(0)$ , in a similar way as the above description of the polydisc itself.
- (b) Let  $\delta = (\delta_1, \delta_2)$  with  $\delta_1, \delta_2 > 0$ . Let  $(a, b) \in \partial B_{\epsilon}(0)$ . Show that the polydisc  $B_{\delta}(a, b)$  around (a, b) contains both a point in  $B_{\epsilon}(0)$  and a point in the complement of  $B_{\epsilon}(0)$  by explicitly writing two such points.

Solution:

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**Q-2)** Let U be a non-empty open subset in  $\mathbb{C}^n$  where n > 1, and let  $f : U \to \mathbb{C}$  be a non-constant holomorphic function. Show that f(U) is open in  $\mathbb{C}$ . You may use the fact that the statement is true when n = 1.

Solution:

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**Q-3)** Let U be a non-empty open subset in  $\mathbb{C}^n$  where n > 1, and let  $f : U \to \mathbb{C}$  be a non-constant holomorphic function. Show that there is no point  $z_0 \in U$  such that  $|f(z_0)| \ge |f(z)|$  for all  $z \in U$ . You may use the fact that the statement is true when n = 1.

Solution: