Due Date: 12 May 2015, Tuesday
Time: Class time
Instructor: Ali Sinan Sertöz

NAME:..

STUDENT NO:.......................................

Math 430 / Math 505 Introduction to Complex Geometry – Midterm Exam II

1	2	3	4	5	TOTAL
40	30	30	0	0	100

Please do not write anything inside the above boxes!

Check that there are **3** questions on your exam booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit. **Submit your solutions on this booklet only. Use extra pages if necessary.**

Rules for Homework and Take-Home Exams

(1) You may discuss the problems with your classmates or with me but it is absolutely mandatory that you write your answers alone. Any similarity with your written words with any other solution or any other source that I happen to know is a direct violation of honesty.

(2) In particular do not lend your written solutions to your friends, nor borrow your friends's written solutions. Oral exchange of ideas is acceptable and is in fact encouraged.

(3) You must obey the usual rules of attribution: all sources you use must be explicitly cited in such a manner that the source can be easily retrieved by the reader. This includes any ideas you borrowed from your friends.

(4) Finally, in your written solution make sure that you exhibit your total understanding of the ideas involved, even mentioning where you quote a result but don't really follow the reasoning. This is an essential ingredient of learning.

Affidavit of compliance with the above rules: *I affirm that I have complied with the above rules in preparing this submitted work. Every solution I wrote reflects my true understanding of the problem. Any sources used, ideas from friends or others are explicitly cited without exception.*

Please sign here:

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Q-1) Prove the Cauchy Integral Formula for smooth functions on an annulus. That is, show that

$$f(z) = \frac{1}{2\pi i} \int_{\partial A_r} \frac{f(w)}{w-z}\, dw + \frac{1}{2\pi i} \int_{A_r} \frac{\partial f(w)}{\partial \bar{w}} \frac{dw \wedge d\bar{w}}{w-z}, \quad \text{for all} \quad z \in A_r,$$

where $0 < r < 1$, $A_r = \{z \in \mathbb{C} \mid r < |z| < 1\}$ and $f \in C^\infty(\bar{A}_r)$.
Also show how this formula reduces to the usual one when f is C^∞ in the closed unit disk.

Solution:

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Q-2) Prove the $\bar{\partial}$-Poincare lemma for the punctured disk. That is, for any $g \in C^\infty(\Delta^*)$, there exists an $f \in C^\infty(\Delta^*)$ such that

$$\frac{\partial f(z)}{\partial \bar{z}} = g(z), \quad \text{for} \quad z \in \Delta^*,$$

where

$$\Delta^* = \{z \in \mathbb{C} \mid 0 < |z| < 1\}.$$

Solution:

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Q-3) Show that every holomorphic line bundle on the unit punctured disc in the plane is trivial.

Solution: