Due Date: 14 April 2017, Friday
Time: Class time
Instructor: Ali Sinan Sertöz

NAME:..

STUDENT NO:..

Math 430 / Math 505 Introduction to Complex Geometry – Homework 4 – Solutions

1	2	3	4	5	TOTAL
50	50	0	0	0	100

Please do not write anything inside the above boxes!

Check that there are **2** questions on your booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.
Submit your solutions on this booklet only. Use extra pages if necessary.

Rules for Homework and Take-Home Exams

(1) You may discuss the problems with your classmates or with me, or even with people who took this course before. It is not considered good behavior to ask these questions at online forums without mentioning that these are homework questions of an introductory course.

(2) You may use any written source be it printed or online. Google search is perfectly acceptable.

(3) **It is absolutely mandatory that you write your answers alone.**

(4) You must obey the usual rues of attribution: all sources you use must be explicitly cited in such a manner that the source is easily retrieved with your citation. This includes any ideas you borrowed from your friends. *(It is always nice to flatter your friends by using their ideas and thanking them.)*

(5) Even if you find a solution online, you must rewrite it in your own narration, fill in the blanks if any, making sure that you exhibit your total understanding of the ideas involved.

(6) Do not lend your written work to your friends and do not ask to borrow their written work. You may explain your solutions to your friends to any degree of detail you like, or you may ask them as many questions as they are willing to answer. **But the final writing process should be done alone.**

Affidavit of compliance with the above rules: I affirm that I have complied with the above rules in preparing this submitted work. Moreover I declare that every solution I wrote reflects my true understanding of the problem, and any sources used, including ideas from friends, are explicitly cited without exception.

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Q-1) Let N and M be complex manifolds of dimensions n and m respectively. Let ϕ be a (p,q)-form on M, i.e. $\phi \in A^{(p,q)}(M)$. Consider a map $f\colon N \to M$. Show that

(a) $f^*\phi \in \oplus_{s+t=p+q} A^{(s,t)}(N)$ if f is C^∞.

(b) $f^*\phi \in A^{(p,q)}(N)$ if f is holomorphic.

Solution:

Let p be any point in N and let $(x_1, \ldots, x_n)$ and $(w_1, \ldots, w_m)$ be local coordinates on N and M respectively, around p and $f(p)$. Moreover let $f(x) = (f_1(x), \ldots, f_m(x))$.

Notice that $f^*(dw_j) = d(w_j(f)) = df_j = \sum_{k=1}^{n} \left(\frac{\partial f_j}{\partial x_k} \, dx_k + \frac{\partial f_j}{\partial \bar{x}_k} \, d\bar{x}_k \right)$. And also $f^*(d\bar{w}_j) = d(\bar{w}_j(f)) = d\bar{f}_j = \sum_{k=1}^{n} \left(\frac{\partial \bar{f}_j}{\partial x_k} \, dx_k + \frac{\partial \bar{f}_j}{\partial \bar{x}_k} \, d\bar{x}_k \right)$.

Now it is clear that the claim of the first part is true. But if f is holomorphic, then $\frac{\partial f_j}{\partial \bar{x}_k} = 0$ and $\frac{\partial \bar{f}_j}{\partial x_k} = 0$, so the claim of the second part is also valid.

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Q-2) Let M be a complex manifold of dimension n, and let A and B be analytic subvarieties of complementary dimensions intersecting transversally at $p \in A \cap B$. Show that the intersection index $\iota_p(A \cdot B)$ is always $+1$ when we give M, A, B the natural orientations imposed by their complex structures.

Solution:

Suppose M has coordinates $z_1, \ldots, z_n$ around p. Then the orientation of T_pM' is given by

$$\frac{\partial}{\partial x_1}, \frac{\partial}{\partial y_1}, \ldots, \frac{\partial}{\partial x_n}, \frac{\partial}{\partial y_n},$$

where $z_k = x_k + iy_k$. Interchanging z_k with z_ℓ involves moving the pairs $\frac{\partial}{\partial x_k}, \frac{\partial}{\partial y_k}$ and $\frac{\partial}{\partial x_\ell}, \frac{\partial}{\partial y_\ell}$ around. This does not change the orientation.