Due Date: 14 February 2012, Tuesday NAME:..

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Math 431 Algebraic Geometry – Homework – Solutions

1	2	3	4	TOTAL
10	10	10	10	40

Please do not write anything inside the above boxes!

Notes: This homework is about the first chapter of the textbook. Almost all the answers are there. Read the chapter and write your answers in your own words in a way to convey the impression that you understood what is involved.

Q-1) On page 2, the author says "...which are accurate *topological* pictures of the curves but which do not reflect the way they sit inside $\mathbb{C}^2$." Explain what she means. Give an example (Hint: There are some examples a few lines below!)

Answer:

The actual shape of the curves would be be pictured using biholomorphic maps but we use homeomorphic maps which change the shape without tearing them. Our objects lie in 4 dimensional real space and we try to imagine how they would look like if we pushed them into three space using only homeomorphisms. This way we get some idea about their topological properties but not about their actual shape.

An example is given by the complex curves $xy = 0$. This represents a real surface in $\mathbb{R}^4$. It consists of two planes in $\mathbb{R}^4$ touching each other at the origin without being tangent to each other and without acquiring a singularity there . There is no way to see this in $\mathbb{R}^3$.

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Q-2) What is Fields Medal in mathematics? List all people whose Fields Medal citations referred to their work in algebraic geometry. (This is not in the book. Some Googling is required. No copy-pasting will be tolerated!)

Answer:

Fields Medal is the prestigious prize awarded by The International Mathematical Union (IMU) every four years. See their page on Fields Medal:
http://www.mathunion.org/general/prizes/fields/details/

See also Encyclopedia Britannica online for a list of Field medallists and their primary research area:
http://www.britannica.com/EBchecked/topic/206375/Fields-Medal

Here is a list compiled from the latter web page, giving the names of Fields medallists whose work in algebraic geometry is quoted for the medal.

1 1954, Kodaira Kunihiko

2 1966, Grothendieck, Alexandre

3 1970, Hironaka Heisuke

4 1974, Mumford, David

5 1978, Deligne, Pierre

6 1990, Drinfeld, Vladimir

7 1990, Mori Shigefumi

8 2002, Voevodsky, Vladimir

9 2010, Ngo Bao Chau

However the list can be extended. For example Gerd Faltings received the Medal in 1986 for his solution of the Mordell conjecture which is a problem in arithmetic algebraic geometry.

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Q-3)

i Section 1.2.1 end with the words "Whether that finite number is ever nonzero, nobody knows."

What is that number? How does she conclude that it is finite? The book was written in 1992. Is there any progress on this topic since then?

ii The author claims that "... singularities [of curves] are important in many parts of mathematics..." What examples does she mention? How does she relate the existence of singularity on a curve with the existence of tangents?

iii The author gives two examples to show that the theory of real curves is different than the theory of complex curves. What are these examples?

Answer:

i This is the number of rational points on the complex curve $s^n + t^n = 1$, or equivalently the number of nontrivial integer solutions of the Fermat equation $x^n + y^n = z^n$ when $n \geq 3$. The $n = 3$ case is known since Euler. Kummer proved this for certain prime values of n. But the general case remained open until Faltings proved that this number must be finite. He received a Fields for this work. Then in 1995 Andrew Wiles settled the problem by showing that the number of non-trivial solutions of the Fermat equation is zero. He was barely over 40 years old when he finished his proof, so the International Mathematical Union awarded him with a silver plate instead of a Fields Medal in 1998.

ii She mentions knot theory and catastrophe theory. The singularities are points where a unique tangent to the curve does not exist.

iii These examples are given on pages 6 and 7. The first example shows that you get a better classification pattern of curves with complex numbers. The other example is about certain polynomials not having real roots but always having complex roots.

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Q-4)

i What is a conchoid and how do we use it to trisect an angle?

ii What is a cissoid and how can we use it to solve the Delian problem?

iii What is a cycloid and which famous problem does it solve?

Answer:

i Conchoid is defined on page 25. The conchoid of Nicomedes, which is used in trisecting an angle is defined on page 26. And finally the figure associated to this trisection is Figure 1.15 on page 28.

ii The so called Delian problem is the problem of doubling a cube. This is all explained on page 28.

iii Cycloid is explained briefly on page 26. It solves the brachistochrone and the tautochrone problems. Look them up.