Due Date: 29 March 2012, Thursday NAME:..

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Math 431 Algebraic Geometry – Homework – Solutions

previous	7	8	9	10	TOTAL
60	10	10	10	10	100

Please do not write anything inside the above boxes!

Q-7) Let $\phi : C \to \mathbb{P}^2$ be defined by $\phi[x : y : z] = [x : z]$ where C is a nonsingular projective curve in the projective plane not containing the point $[0 : 1 : 0]$. Show that if C has degree $d > 1$, then ϕ has at least one ramification point. Show that if $d = 1$, then ϕ has no ramification points and is a homeomorphism.

Answer:

Let $C = Z(P)$ where P is homogeneous of degree d. The ramification points are $Z(P) \cap Z(P_y)$ which is nonempty when $d > 1$.

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Q-8) Show that the projective curve D defined by $y^2z = x^3$ has a unique singular point. Show that the map $f : \mathbb{P}^1 \to D$ defined by

$$f[s : t] = [s^2t : s^3 : t^3]$$

is a homeomorphism. Deduce that the degree-genus formula cannot be applied to singular curves in $\mathbb{P}^2$.

Answer:

$f^{-1}[x : y : z] = [y : x] = [\sqrt{x} : \sqrt{z}]$ after choosing a branch.

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Q-9) Let C be a singular irreducible projective cubic curve in $\mathbb{P}^2$. Show that the tangent line to C at a nonsingular point or a line through two distinct nonsingular points of C cannot meet C at a singular point.

Answer: Use Bezout theorem on page 52 together with lemma 2.24 on page 63.

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Q-10) Show that if p is a point of inflection on a nonsingular cubic curve C in $\mathbb{P}^2$, then there are exactly four tangent lines to C which pass through p.

Answer: We can make change of variables so that the inflection point is $p = [0 : 1 : 0]$ and the curve is given by

$$f(x, y, z) = y^2 z - x(x - z)(x - \lambda z), \text{ where } \lambda \in \mathbb{C} - \{0, 1\}.$$

Any line through p is of the form $Ax + Bz = 0$ where $(A, B) \neq (0, 0)$.

For any point $q \in C$, the tangent line is of the form $\nabla f(q) \cdot (x, y, z) = 0$. We observe that $\nabla f(q) = (*, 0, *)$ when q is one of the following four points.

$$[0 : 0 : 1], [1 : 0 : 1], [\lambda : 0 : 1], [0 : 1 : 0].$$