Due Date: 19 April 2012, Thursday NAME:..

Ali Sinan Sertöz STUDENT NO:..

Math 431 Algebraic Geometry – Homework – Solutions

previous	11	12	13	TOTAL
100	10	10	10	130

Please do not write anything inside the above boxes!

Q-11) Let R and S be connected Riemann surfaces with R compact. Show that every holomorphic map $f : R \to S$ is surjective. Show how it follows from this that there are no non-constant holomorphic functions on a connected compact Riemann surface.

Answer:

Since f is a non-constant holomorphic map, it is an open mapping. Hence $f(R)$ is an open subset of S.

On the other hand since f is continuous, $f(R)$ is compact. Being a compact subset of a compact set, $f(R)$ is closed.

$f(R)$ being both open and closed, and non-empty, must be the whole of S, where we assume that Riemann surfaces are defined to be irreducible and connected.

If f is a holomorphic function on a compact Riemann surface then it is a holomorphic function to $\mathbb{P}^1$ which is not onto, since it does not take ∞. By the above argument it must be constant.

NAME: STUDENT NO:

Q-12) Let Λ be a lattice in $\mathbb{C}$ generated by ω_1 and ω_2. Define

$$g_2(\Lambda) = 60 \sum_{\omega \in \Lambda - \{0\}} \omega^{-4}, \quad \text{and} \quad g_3(\Lambda) = 140 \sum_{\omega \in \Lambda - \{0\}} \omega^{-6}.$$

Show that

$$g_2(\Lambda)^3 - 27 g_3(\Lambda)^2 \neq 0.$$

Answer:

On page 120, the curve C_Λ is defined as the set of points $[x : y : z] \in \mathbb{P}^2$ satisfying the equation

$$y^2 z = 4x^3 + g_2 x z^2 - g_3 z^3.$$

Lemma 5.20 on page 120 says that this curve is nonsingular. In particular it is smooth on the chart $z = 1$. This means that the cubic

$$4x^3 - g_2 x - g_3$$

has no repeated roots. Therefore its discriminant, which is $g_2^3 - 27 g_2^2$, is non-zero.

NAME: STUDENT NO:

Q-13) Let Λ, $g_2(\Lambda)$ and $g_3(\Lambda)$ be as in the previous question. Let C_Λ be the projective curve in $\mathbb{P}^2$ defined by

$$y^2z = 4x^3 - g_2(\Lambda)xz^2 - g_3(\Lambda)z^3.$$

Once the curve C_Λ is known, described how to recover the lattice Λ.

Answer:

This is the content of Corollary 6.18 on page 151.