Due Date: May 26, 2014 Monday, 17:30 NAME:..

Instructor: Ali Sinan Sertöz STUDENT NO:..

Math 431 Algebraic Geometry – Final Exam

1	2	3	4	TOTAL
25	25	25	25	100

Please do not write anything inside the above boxes!

Check that there are **4** questions on your booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.

NAME: STUDENT NO:

Q-1) Let

$$H = k + kt^{4\nu}(1+t) + kt^{6\nu}(1+t) + kt^{7\nu}(1+t) + k[[t]]t^{8\nu},$$
$$H' = k + kt^{4\nu}(1+t+t^2) + kt^{6\nu}(1+t+t^2) + kt^{7\nu}(1+t+t^2) + k[[t]]t^{8\nu}$$

where $\nu > 2$. Show that these two rings are both Arf rings, have the same characters but are not isomorphic.

Solution:

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Q-2) For an effective divisor $D \geq 0$, on a curve X, define

$$|D| = \{D' \in \mathrm{Div}(X) \bigm| D' \geq 0 \text{ and } D' \sim D\},$$

where $D' \sim D$ means that there exists a rational function f on X such that $D' = D + (f)$.

(i) Show that $|D|$ is isomorphic to $\mathbb{P}^{\ell}$, where $\ell = \ell(D) - 1$.

(ii) For two effective divisors $D \geq 0$ and $E \geq 0$, show that

$$\dim|D| + \dim|E| \leq \dim|D + E|.$$

(iii) Prove Clifford's theorem that if D is an effective divisor such that $K - D$ is also effective, where K is the canonical divisor of X, then

$$\ell(D) \leq \frac{1}{2}\deg D + 1.$$

Solution:

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Q-3) Assume that the projection $\pi : \mathbb{P}^n \times \mathbb{P}^m \to \mathbb{P}^m$ is a closed map, i.e. it maps closed sets to closed sets (in the Zariski topology). Let $f : X \to Y$ be a morphism of projective varieties, $X \subset \mathbb{P}^n$, $Y \subset \mathbb{P}^m$. Let $\Delta(Y) = \{(y, y) \in Y \times Y\}$ be the diagonal. Show that $\Delta(Y)$ is closed in $\mathbb{P}^m \times \mathbb{P}^m$. Let $\Gamma_f = \{(x, f(x)) \in X \times Y\}$ be the graph. Show that Γ_f is closed in $\mathbb{P}^n \times \mathbb{P}^m$. Now show that $f(X)$ is closed in Y. In particular show that if X and Y are curves (smooth and irreducible), then $f(X)$ is either Y or a point.

Solution:

NAME: STUDENT NO:

Q-4) Show that every projective algebraic set can be written as the zero set of finitely many homogeneous polynomials all of the same degree.

Solution: