Math 431 Algebraic Geometry – Midterm Exam 2

1	2	3	4	TOTAL
50	50	-	-	100

Please do not write anything inside the above boxes!

Check that there are **2** questions on your booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.

Q-1) Let H be a subring of $k[[t]]$ which contains all formal sums of its elements. Let $W(H) = \{i_0, i_1, i_2, \dots\}$ be the semigroup of orders of elements in H, where we have $0 = i_0 < i_1 < i_2 < \cdots$. Show that for <u>any</u> choice of elements $S_{i_0}, S_{i_1}, S_{i_2}, \dots$ of elements of H with $\operatorname{ord} S_{i_\ell} = i_\ell$, we have

$$H = \{\sum_{\ell=0}^{\infty} \alpha_\ell S_{i_\ell} \mid \alpha_\ell \in k\}.$$

Answer:

Q-2) For any fixed positive integer r, choose elements $T_1, \ldots, T_r \in k[[t]]$ such that $\operatorname{ord} T_r > 0$ and

$$T_i \in kT_{i+1} + kT_{i+1}T_{i+2} + \cdots + kT_{i+1}\cdots T_{r-1} + k[[t]]T_{i+1}\cdots T_r,$$

for $i = 1, \ldots, r-1$.

Show that the ring

$$k + kT_1 + kT_1T_2 + \cdots + kT_1\cdots T_{r-1} + k[[t]]T_1\cdots T_r$$

is an Arf ring and moreover every Arf ring is of this form.

Solution: