Homework-4 for Math 431 Due Date: 22 April 2022 Friday

Let $f \in k[x_0, \ldots, x_n]$ be irreducible homogeneous polynomials. Show that $\mathbb{A}^n - Z(f)$ is affine, i.e. isomorphic to $\mathbb{A}^n$.

Write your solutions on A4 papers and staple them together. Do not use other fancy covers!

Thank you.

Solution:
First let $H \subset \mathbb{P}^n$ be a hyperplane. By change of variables we can assume that $H = Z(x_0)$. Then $\mathbb{P}^n - H = U_0 \cong \mathbb{A}^n$ which is affine.

Next let H be a hypersurface of degree d. The d-uple embedding ν_d sends H to a hyperplane H' in $\mathbb{P}^N$, and hence $\mathbb{P}^N - H'$ is affine. Since $\nu_d(\mathbb{P}^n - H) = \nu_d(\mathbb{P}^n) \cap (\mathbb{P}^N - H')$, and ν_d is an isomorphism onto its image, $\mathbb{P}^n - H$ is also affine.