Due Date: 15 November 2012, Thursday NAME:..

Ali Sinan Sertöz STUDENT NO:..

Math 503 Complex Analysis – Exam 04

1	2	3	4	5	TOTAL
50	50	0	0	0	100

Please do not write anything inside the above boxes!

Check that there are **2** questions on your exam booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.

NAME: STUDENT NO:

Q-1) Let $\mathcal{F} \subset H(G)$ be a locally bounded family. Show that:

> For every $p \in G$ and for every $\epsilon > 0$, there exists a $\delta > 0$ such that for every $f \in \mathcal{F}$ and for every $x, y \in B_\delta(p)$, we have $|f(x) - f(y)| < \epsilon$.

Solution:

By Montel's theorem $\mathcal{F}$ is normal. By Arzelo-Ascoli, $\mathcal{F}$ is equicontinuous at every point of G.

Here is a direct proof.

Choose any $p \in G$ and any $\epsilon > 0$. Since $\mathcal{F}$ is locally bounded, there exist constants $M, r > 0$ such that for every $f \in \mathcal{F}$ and for every $z \in B_r(p)$, we have $|f(z)| < M$.

Now choose a $\delta > 0$ such that $0 < \delta < \min\{r/2, (r\epsilon)/(4M)\}$. Using Cauchy Integral Formula, we have for every $f \in \mathcal{F}$ and for every $x, y \in B_\delta(p) \subset B_{r/2}(p)$,

$$
\begin{aligned}
|f(x) - f(y)| &= \left| \frac{1}{2\pi i} \int_{|z-p|=r/2} \frac{f(z)\,dz}{z-x} - \frac{1}{2\pi i} \int_{|z-p|=r/2} \frac{f(z)\,dz}{z-y} \right| \\
&= \frac{1}{2\pi} \left| \int_{|z-p|=r/2} \frac{f(z)(x-y)\,dz}{(z-x)(z-y)} \right| \\
&< \frac{1}{2\pi} \frac{(M)(2\pi(r/2))(2\delta)}{(r/2)^2} \\
&= \frac{4M\delta}{r} \\
&< \epsilon,
\end{aligned}
$$

as required.

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Q-2) Check Weierstrass Factorization Theorem on page 170 of Conway's book, *Functions of One Complex Variable*, and explain where we need any of the information we studied on Monday about the metric space $H(G)$.

Solution:

Notice that the proof of Weierstrass theorem rests heavily on showing the convergence of the sequence of partial products. Nowhere in the proof you check that the limit is an analytic function. It is taken care of by the fact that $H(G)$ is a complete metric space, see Corollary 2.3 on page 152, in Conway's book.