Due Date: 30 October 2014, Thursday – Class time NAME:..

Ali Sinan Sertöz STUDENT NO:..

Math 503 Complex Analysis – Homework 2 – Solutions

1	2	3	4	5	TOTAL
50	50	0	0	0	100

Please do not write anything inside the above boxes!

Check that there are **2** questions on your exam booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.

NAME: STUDENT NO:

Q-1) Let $S = S_{(a,b,c)}$ be the unit sphere in $\mathbb{R}^3$ centered at (a, b, c) with $c > -1$. Let $\alpha = a + ib$. Find a rigid motion T of S such that

$$P_{T(S)} \circ T \circ P_S^{-1}(z) = \frac{1}{z - \alpha},$$

where P_S denotes the stereographic projection to $\mathbb{C}$ from the North pole of the sphere S, and similarly for $P_{T(S)}$.

Solution:

Here we use (x, y, t) as the Cartesian coordinates in $\mathbb{R}^3$, and by $P_{\vec{p}}$ we mean the stereographic projection to the complex plane $t = 0$ from the North pole of the unit sphere centered at $\vec{p}$. In particular we have

$$P_{(a,b,c)}(x, y, t) = \left(\frac{(1 + c)x - at}{1 + c - t}, \ \frac{(1 + c)y - bt}{1 + c - t} \right),$$

where $(x, y, t) \in S_{(a,b,c)}$, and $S_{(a,b,c)}$ is the unit sphere centered at (a, b, c).

Start with the unit sphere whose center is at (a, b, c). Move it to the origin, rotate around x-axis by π radians, move the center down to $(0, 0, -c/(1 + c))$. The resulting rigid motion corresponds to

$$\frac{1}{z - \alpha} \quad \text{where} \quad \alpha = a + ib.$$

Writing

$$P^{-1}_{(a,b,c)}(X, Y) = (x_0, y_0, t_0),$$

we have

$$\frac{1}{z - \alpha} = P_{(0,0,-\frac{c}{1+c})}(x_0 - a, -y_0 + b, -t_0 + \frac{c^2}{1 + c}).$$

NAME: STUDENT NO:

Q-2) Start with the unit sphere S centered at $(0,0,k)$ with $k > -1$. Let $r > 0$ be a real number. Find a rigid motion T of S such that

$$P_{T(S)} \circ T \circ P_S^{-1}(z) = rz.$$

Solution:

We use the notation of the previous solution.

Move the sphere S along the t-axis by s. The overall effect is

$$P_{(0,0,k+s)}(P_{(0,0,k)}^{-1}(X,Y) + (0,0,s)) = \frac{1+k+s}{1+k}(X,Y).$$

We therefore solve for s from the equation

$$r = \frac{1+k+s}{1+k},$$

which gives

$$s = (k+1)(r-1).$$