

Due Date: 24 November 2014, Monday – Class time      NAME:.....

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**Math 503 Complex Analysis – Homework 3**

1	2	3	4	5	TOTAL
50	25	25	0	0	100

*Please do not write anything inside the above boxes!*

Check that there are **3** questions on your exam booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.

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**Q-1** For any  $a \in D = \{|z| < 1\}$ , we define

$$\phi_a(z) = \frac{z - a}{1 - \bar{a}z}, \quad \text{for } z \in D.$$

We know that  $\phi_a(D) = D$ . Show that for any  $a, b \in D$ , there exists  $c \in D$  such that

$$\phi_a \circ \phi_b = \lambda \phi_c,$$

where  $\lambda$  is a complex number with  $|\lambda| = 1$ . (Make sure to check that  $|c| < 1$ .)

Moreover let  $\alpha \in \partial D$ , i.e.  $|\alpha| = 1$ . Show that there exist  $d \in D$  and  $\beta \in \partial D$  such that

$$\phi_a(\alpha \phi_b(z)) = \beta \phi_d(z) \quad \text{for all } z \in D.$$

(Again check that  $|d| < 1$  and  $|\beta| = 1$ .)

**Solution:**

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**Q-2)** [Conway, p133, Exercise 7] Suppose that  $f$  is analytic in a region containing the closure of  $D = \{|z| < 1\}$ . Assume that  $|f(z)| < 1$  for  $z \in D$ . Assume further that  $f$  has a simple zero at  $\frac{1}{4}(1+i)$  and a double zero at  $\frac{1}{2}$ . Can  $f(0) = \frac{1}{2}$ ?

**Solution:**

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**Q-3)** [Conway, p133, Exercise 8] Is there an analytic function  $f$  on  $D = \{|z| < 1\}$  such that  $|f(z)| < 1$  for  $|z| < 1$ ,  $f(0) = \frac{1}{2}$ , and  $f'(0) = \frac{3}{4}$ ? If so, find such an  $f$ . Is it unique?

**Solution:**