

Due Date: 29 December 2014, Monday – Class time NAME:.....

Ali Sinan Sertöz STUDENT NO:.....

Math 503 Complex Analysis – Homework 5

1	2	3	4	5	TOTAL
100	0	0	0	0	100

Please do not write anything inside the above boxes!

Check that there is **1** question on your exam booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.

NAME:

STUDENT NO:

Q-1) Prove the following formula for $\operatorname{Re} z > 0$.

$$\Gamma(z) \frac{\sin \theta z}{n(a^2 + b^2)^{z/2}} = \int_0^\infty e^{-at^n} t^{nz-1} \sin(bt^n) dt$$

where n is a positive integer, a and b are real numbers with $(a, b) \neq (0, 0)$, and $\tan \theta = b/a$. When $a = 0$, we take $\theta = \pm\pi/2$ such that $\theta b > 0$.

[Hint: Start with $\Gamma(z) = \int_0^\infty e^{-s} s^{z-1} ds$ and make the substitution $s = (a + ib)t^n$.]

Using this formula evaluate

$$\int_0^\infty \sin t^n dt.$$

Solution: