Due Date: 17 November 2014, Monday – Class time NAME:..

Ali Sinan Sertöz STUDENT NO:..

Math 503 Complex Analysis – Take-Home Midterm Exam 1 –

1	2	3	4	TOTAL
20	20	20	40	100

Please do not write anything inside the above boxes!

Check that there are **4** questions on your exam booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.

NAME: STUDENT NO:

Q-1) For a fixed integer $n > 0$ and a fixed real number $\alpha > 0$, find all entire functions f satisfying $|f(z)| \leq \alpha|z|^n$ for all $z \in \mathbb{C}$.

Solution:

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Q-2) Note that $\operatorname{cotan} z$ is a meromorphic function with a simple pole at each $z = \pi n$, where $n \in \mathbb{Z}$. Therefore its Laurent series

$$\operatorname{cotan} z = \frac{b_1}{z} + \sum_{n=0}^{\infty} a_n z^n$$

converges for $|z| < \pi$. Determine the coefficients $b_1, a_0, a_1, \ldots, a_n, \ldots$.
The standard and easiest way to do this to use the following facts:
(a) $e^{iz} = \cos z + i \sin z$, for all $z \in \mathbb{C}$, and
(b) $\dfrac{z}{e^z - 1} = \displaystyle\sum_{n=0}^{\infty} \frac{B_n}{n!}\, z^n$, for $|z| < 2\pi$, where B_n are Bernoulli numbers with the convention that $B_0 = 1$ and $B_1 = -\dfrac{1}{2}$.

Solution:

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Q-3) Let U be a non-empty, open and connected subset of $\mathbb{C}$, and let f be a holomorphic map on U. Assume that there is a point $z_0 \in U$ such that $|f(z_0)| \geq |f(z)|$ for all $z \in U$.

1. Using Cauchy Integral Formula, show that $|f(z)| = c$, a constant, for all $z \in U$.
2. Using Cauchy-Riemann equations, show that f is constant, assuming that $|f(z)|$ is constant.
3. Using the Open Mapping Theorem, show that f is constant, assuming that $|f(z)|$ is constant.

Solution:

NAME: STUDENT NO:

Q-4) For any $\alpha \in \mathbb{R}$, define the integral

$$I_\alpha = \int_0^\infty \frac{\log(1+x^2)}{x^{1+\alpha}} \, dx.$$

Show that I_α exists if and only if $0 < \alpha < 2$, and in that case we have

$$I_\alpha = \frac{\pi}{\alpha} \ \operatorname{cosec}(\frac{\pi}{2} \alpha).$$

Solution: