Due Date: 13 October 2016, Thursday
Class Time

NAME:.....

STUDENT NO:.....

**Math 503 Complex Analysis - Homework 1**

| 1 | 2 | 3 | 4 | TOTAL |
|---|---|---|---|---|
| | | | | |
| 25 | 25 | 25 | 25 | 100 |

*Please do not write anything inside the above boxes!*

Check that there are **4** questions on your booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.
**Submit your solutions on this booklet only. Use extra pages if necessary.**

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**Rules for Homework Assignments**

(1) You may discuss the problems with your classmates or with me but it is absolutely mandatory that you **write your answers alone**.

(2) You must obey the usual rules of attribution: all sources you use must be explicitly cited in such a manner that the source is easily retrieved with your citation. This includes any ideas you borrowed from your friends. (It is a good thing to borrow ideas from friends but it is a bad thing not to acknowledge their contribution!)

(3) Even if you find a solution online, you must rewrite it in your own narration, fill in the blanks if any, making sure that you **exhibit your total understanding of the ideas involved**.

**Affidavit of compliance with the above rules:** I affirm that I have complied with the above rules in preparing this submitted work.

*Please sign here:*

NAME: STUDENT NO: DEPARTMENT:

**Q-1)** Let $\Lambda$ be a circle lying on the unit sphere $S = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1^2 + x_2^2 + x_3^2 = 1\}$. Show that the stereographic projection of $\Lambda$ to $\mathbb{C}$ is a straight line if $\Lambda$ passes through the North pole, and that it is a circle otherwise.

**Solution:**

NAME: STUDENT NO: DEPARTMENT:

**Q-2)** Show that $\{\operatorname{cis} k \mid k = 0, 1, 2, \dots\}$ is dense in $T = \{z \in \mathbb{C} \mid |z| = 1\}$.

**Solution:**

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**Q-3** Let $\{f_n\}$ be a sequence of uniformly continuous functions from a metric space $(X, d)$ into another metric space $(Y, p)$ and suppose that $f = u - \lim f_n$ exists. Prove that $f$ is uniformly continuous.

**Solution:**

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**Q-4)** Let $f, g, h : \mathbb{R} \to \mathbb{R}$ be $C^2$-functions with $f(0) = 1$ and $h(0) = 0$. Moreover assume that the complex function

$$\phi(x + iy) = \sin x \cdot f(y) + ig(x)h(y)$$

is analytic on $\mathbb{C}$. (Here $x$ and $y$ are real variables.) Find $f, g, h$ explicitly.

**Solution:**