Due Date: 8 December 2016,
Thursday, Class Time

NAME:.....

STUDENT NO:.....

Math 503 Complex Analysis - Homework 3

1	2	3	4	TOTAL
25	25	25	25	100

Please do not write anything inside the above boxes!

Check that there are **4** questions on your booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.
Submit your solutions on this booklet only. Use extra pages if necessary.

Rules for Homework Assignments

(1) You may discuss the problems with your classmates or with me but it is absolutely mandatory that you **write your answers alone**.

(2) You must obey the usual rules of attribution: all sources you use must be explicitly cited in such a manner that the source is easily retrieved with your citation. This includes any ideas you borrowed from your friends. (It is a good thing to borrow ideas from friends but it is a bad thing not to acknowledge their contribution!)

(3) Even if you find a solution online, you must rewrite it in your own narration, fill in the blanks if any, making sure that you **exhibit your total understanding of the ideas involved**.

Affidavit of compliance with the above rules: I affirm that I have complied with the above rules in preparing this submitted work.

Please sign here:

NAME: STUDENT NO: DEPARTMENT:

Q-1) Construct a sequence of functions $f_n(x) : \mathbb{R} \to \mathbb{R}$ such that each $f_n(x)$ is real analytic, i.e. at every $x_0 \in \mathbb{R}$ each $f_n(x)$ has a Taylor expansion converging to the function itself, and moreover $f_n(x)$ converges uniformly to $f(x)$ on $\mathbb{R}$ where $f(x) = |x|$.
Note that the uniform limit of these real analytic functions is not analytic. This never happens with complex analytic functions.

Solution:

NAME: STUDENT NO: DEPARTMENT:

Q-2) Let D be the unit disc. Find all analytic functions $f : D \to D$ with at least two fixed points.

Solution:

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Q-3

(a) Does there exist an analytic surjective map $f\colon D^* \to D$? Here D is the unit disc around the origin and D^* is D with the origin removed.

(b) Does there exist an analytic surjective map $f\colon D \to D^*$?

Solution:

NAME: STUDENT NO: DEPARTMENT:

Q-4) For any positive integer n calculate the integral

$$I_n = \int_0^\infty \frac{dx}{(x^2+1)^n}.$$

Solution: