Due Date: 27 October 2016, Thursday
Class Time

NAME:..

STUDENT NO:.......................................

Math 503 Complex Analysis - Midterm Exam 1

1	2	3	4	TOTAL
25	25	25	25	100

Please do not write anything inside the above boxes!

Check that there are **4** questions on your booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.
Submit your solutions on this booklet only. Use extra pages if necessary.

Rules for Take-Home Assignments

(1) You may discuss the problems with your classmates or with me but it is absolutely mandatory that you **write your answers alone**.

(2) You must obey the usual rules of attribution: all sources you use must be explicitly cited in such a manner that the source is easily retrieved with your citation. This includes any ideas you borrowed from your friends. (It is a good thing to borrow ideas from friends but it is a bad thing not to acknowledge their contribution!)

(3) Even if you find a solution online, you must rewrite it in your own narration, fill in the blanks if any, making sure that you **exhibit your total understanding of the ideas involved**.

Affidavit of compliance with the above rules: I affirm that I have complied with the above rules in preparing this submitted work.

Please sign here:

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Q-1) Let a, b, c, d be real numbers and let

$$A(a, b, c, d) = (a + ib)^{c+id} = U(a, b, c, d) + iV(a, b, c, d),$$

where U, V are the real and imaginary parts, and we calculate the principal value. Write explicit real formulas for U and V. Then using a software of your choice (WolframAlpha on the net, for example) evaluate the following using your formula and check that you get the correct values:

(i) $A(1, 2, 3, 4)$

(ii) $A(7, 0, 2, 0)$

(iii) $A(0, -7, -2, 3)$

(iv) $A(0, 7, 2, 3)$

Solution:

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Q-2) Give a description of the Riemann surface of the mapping $w = \dfrac{1}{2}\left(z + \dfrac{1}{z}\right)$, using colors when possible.

Solution:

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Q-3 Every 2×2-matrix A with complex entries defines a Möbius transformation when $\det A \neq 0$, and conversely via the association

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \leftrightarrow \frac{az+b}{cz+d}$$

Here we consider Möbius transformations $T(z) = \dfrac{az+b}{cz+d}$ with $ad - bc = 1$. Define $\alpha(T) = (a+d)$. Prove the following.

1. If two Möbius transformations S and T are conjugate, i.e. there is another Möbius transformation U such that $USU^{-1} = T$, then $\alpha(T)^2 = \alpha(S)^2$. Can we say $\alpha(T) = \alpha(S)$?
2. A Möbius transformation T has exactly one fixed point if and only if $\alpha(T)^2 = 4$.
3. If $\alpha(T) = 4$, then T is conjugate to a translation of the form $z \mapsto z + b$.

Solution:

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Q-4) Let γ be a closed simple rectifiable curve with the origin lying on its interior. Use the fundamental theorem of Calculus to evaluate

$$\int_\gamma \frac{1}{z}.$$

Solution: