

Due Date: 24 November 2016, Thursday
Class Time



NAME:.....

STUDENT NO:.....

Math 503 Complex Analysis - Midterm Exam 2

| 1 | 2 | 3 | 4 | TOTAL |
|----|----|----|----|-------|
| | | | | |
| 25 | 25 | 25 | 25 | 100 |

Please do not write anything inside the above boxes!

Check that there are **4** questions on your booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.

Submit your solutions on this booklet only. Use extra pages if necessary.

Rules for Take-Home Assignments

- (1) You may discuss the problems with your classmates or with me but it is absolutely mandatory that you **write your answers alone**.
- (2) You must obey the usual rules of attribution: all sources you use must be explicitly cited in such a manner that the source is easily retrieved with your citation. This includes any ideas you borrowed from your friends. (It is a good thing to borrow ideas from friends but it is a bad thing not to acknowledge their contribution!)
- (3) Even if you find a solution online, you must rewrite it in your own narration, fill in the blanks if any, making sure that you **exhibit your total understanding of the ideas involved**.

Affidavit of compliance with the above rules: I affirm that I have complied with the above rules in preparing this submitted work.

Please sign here:

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DEPARTMENT:

Q-1 Let f be analytic on $\bar{B}(0; R)$ with $|f(z)| \leq M$ for $|z| \leq R$ and $|f(0)| = a > 0$. Let $\alpha > 1$ be any real number. Show that the number of zeros of f in $B(0; \frac{R}{\alpha})$ is less than or equal to $\frac{1}{\log(\alpha - 1)} \log\left(\frac{M}{a}\right)$.

Solution:

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Q-2 Let f be a non-constant analytic function in $B(0; R)$ and for $0 \leq r < R$ define $A(r) = \max\{\operatorname{Re} f(z) \mid |z| = r\}$. Show that $A(r)$ is a strictly increasing function.

Solution:

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Q-3 Evaluate the improper integral $\int_0^{\infty} \left(\frac{1}{x^2} - \frac{1}{x \sinh x} \right) dx$.

Solution:

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Q-4) Evaluate the following improper integrals.

(a) $\int_0^{\infty} \sin x^2 dx$

(b) $\int_0^{\infty} \cos x^2 dx$

Solution: