Due Date: 5 October 2017, Thursday

NAME:..

STUDENT NO:.......................................

Math 503 Complex Analysis - Homework 1 - Solutions

1	2	3	4	TOTAL
25	25	25	25	100

Please do not write anything inside the above boxes!

Check that there are **4** questions on your booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.
Submit your solutions on this booklet only. Use extra pages if necessary.

General Rules for Take-Home Assignments

(1) You may discuss the problems with your classmates or with me but it is absolutely mandatory that you **write your answers alone**.

(2) You must obey the usual rules of attribution: all sources you use must be explicitly cited in such a manner that the source is easily retrieved with your citation. This includes any ideas you borrowed from your friends. (It is a good thing to borrow ideas from friends but it is a bad thing not to acknowledge their contribution!)

(3) Even if you find a solution online, you must rewrite it in your own narration, fill in the blanks if any, making sure that you **exhibit your total understanding of the ideas involved**.

Affidavit of compliance with the above rules: I affirm that I have complied with the above rules in preparing this submitted work.

Please sign here:

NAME: STUDENT NO: DEPARTMENT:

Q-1) Find the real and imaginary parts of $\left(\frac{1}{2} - i\,\frac{\sqrt{3}}{2}\right)^{2018}$.

Solution: Let $z = \left(\frac{1}{2} - i\,\frac{\sqrt{3}}{2}\right)$. Then $z = \operatorname{cis}(-\frac{\pi}{3})$. Hence

$$z^{2018} = \operatorname{cis}(-\frac{2018 \times 3\pi}{3}) = \operatorname{cis}(-336 \times 2\pi - \frac{2\pi}{3}) = \operatorname{cis}(-\frac{2\pi}{3}) = -\frac{1}{2} - i\,\frac{\sqrt{3}}{2}.$$

Thus

$$\operatorname{Re} z^{2018} = -\frac{1}{2} \quad \text{and} \quad \operatorname{Im} z^{2018} = -\frac{\sqrt{3}}{2}.$$

Also note that

$$z^{2018} = (z^6)^{336} z^2 = z^2 = -\frac{1}{2} - i\,\frac{\sqrt{3}}{2}.$$

NAME: STUDENT NO: DEPARTMENT:

Q-2) Find the real and imaginary parts of $\dfrac{7+i}{(8+i)^2}$.

Solution:

$$\begin{aligned}\frac{7+i}{(8+i)^2} &= \frac{7+i}{63+16i} \\ &= \frac{7+i}{63+16i}\,\frac{63-16i}{63-16i} \\ &= \frac{457-49i}{4225}.\end{aligned}$$

Hence

$$\mathrm{Re}(\frac{7+i}{(8+i)^2}) = \frac{457}{4225} \quad \text{and} \quad \mathrm{Im}(\frac{7+i)}{(8+i)^2}) = -\frac{49}{4225}.$$

NAME: STUDENT NO: DEPARTMENT:

Q-3 Write all cube roots of i in rectangular form, i.e. in the form $a + ib$.

Solution:

The fundamental argument of i is $90°$. Hence a primitive cube root is $\operatorname{cis} 30°$. The other cube roots are then $\operatorname{cis}(30° + 120°)$ and $\operatorname{cis}(30° + 240°)$. Calculating these we find the cube roots as

$$\frac{\sqrt{3}}{2} + i\,\frac{1}{2}, \quad -\frac{\sqrt{3}}{2} + i\,\frac{1}{2}, \quad -i.$$

NAME: STUDENT NO: DEPARTMENT:

Q-4) Let (X, d) be a metric space and $\{x_n\}$ a Cauchy sequence in X. Assume that $\{x_n\}$ has a subsequence $\{x_{n_k}\}$ which converges to some point a in X. Show that $\{x_n\}$ also converges to a.

Solution:

Choose an $\epsilon > 0$. Since $\{x_n\}$ is Cauchy, there exists and index N_1 such that for all $n, m \geq N_1$ we have $d(x_n, x_m) < \epsilon/2$. On the other hand since $\{x_{n_k}\}$ converges to a, there exists and index N_2 such that for all $n_k > N_2$ we have $d(x_{n_k}, a) < \epsilon/2$. Now let N be any index larger than both N_1 and N_2. Choose any $n_k > N$. Then for any $n > N$ we have $d(x_n, a) \leq d(x_n, x_{n_k}) + d(x_{n_k}, a) < \epsilon/2 + \epsilon/2 = \epsilon$, showing that the sequence $\{x_n\}$ also converges to a.