

Due Date: 2 November 2017, Thursday



NAME:.....

STUDENT NO:.....

### Math 503 Complex Analysis - Homework 2

1	2	3	4	TOTAL
25	25	25	25	100

*Please do not write anything inside the above boxes!*

Check that there are **4** questions on your booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.

**Submit your solutions on this booklet only. Use extra pages if necessary.**

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### General Rules for Take-Home Assignments

- (1) You may discuss the problems with your classmates or with me but it is absolutely mandatory that you **write your answers alone**.
- (2) You must obey the usual rules of attribution: all sources you use must be explicitly cited in such a manner that the source is easily retrieved with your citation. This includes any ideas you borrowed from your friends. (It is a good thing to borrow ideas from friends but it is a bad thing not to acknowledge their contribution!)
- (3) Even if you find a solution online, you must rewrite it in your own narration, fill in the blanks if any, making sure that you **exhibit your total understanding of the ideas involved**.

**Affidavit of compliance with the above rules:** I affirm that I have complied with the above rules in preparing this submitted work.

*Please sign here:*

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**Q-1)** Let  $f(z) = u(x, y) + iv(x, y)$  be a complex differentiable function with continuous first partials in a simply connected open set  $G$ . Use Green's theorem to show that

$$\int_{\gamma} f(z) dz = 0$$

for every smooth loop  $\gamma$  in  $G$ .

**Solution:**

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**Q-2)** Let  $f(z) = u(x, y) + iv(x, y)$  be a complex differentiable function with continuous first partials in a simply connected open set  $G$ . Let  $z_0 \in G$  be a fixed point. Show that the integral

$$\int_{|z-z_0|=r} \frac{f(z)}{z-z_0} dz$$

is independent of  $r > 0$  provided that the closed ball of radius  $r$  centered at  $z_0$  totally lies in  $G$ .

**Solution:**

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**Q-3** In the previous question you showed that the value of the given integral is independent of  $r$ . Find that value.

**Solution:**

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**Q-4)** Redo question 1, but this time do not assume the continuity of the first partials. This is Goursat's theorem. For the purposes of this homework, instead of giving all the details of the proof, it suffices to summarize the main ideas used in the proof.

**Solution:**