Bilkent University

Final Exam
Math 503 Complex Analysis I
Due: 31 December 2020
Instructor: Ali Sinan Sertöz

Name & Lastname: ....

Department: ....

Student ID: ....

You may and in fact must discuss your solutions with your friends. Consult any source available. After you ***understand*** the solution start writing your own solution in your own words. When you get stuck, again talk with your friends and consult sources before you continue to write your own solution in your own words.

At the end of each solution quote the sources that you used, including your friends's names who provided you with useful ideas. This is professionalism!

Never borrow your friend's written solution and never lend your written solution.

Then scan and save your solutions as one pdf file and mail it to me before the deadline.

Do not write your solutions assuming that it is my responsibility to decipher your unintelligible handwriting which is spread around the paper carelessly (disrespectful of any potential reader), and on top of that don't expect that I am obliged to figure out what you mean.

That is not how it works.

I am influenced by bad handwriting. I am influenced by careless exposition, and by the lack of any planning. And I don't feel any responsibility to read your mind when you did not explicitly made yourself clear.

My grading is affected by these but grades are not that much important. Later if you ever consider asking me for a reference letter, remember your homework and exam postings. Does any of it merit an reference letter? You be the judge.

Some of you however are presenting excellent solution papers which make it a pleasure to read. You know yourselves. Thank you! ☺

Final Exam Questions

Q-1) Let $f(z) = u(x, y) + iv(x, y)$ be an entire function. Prove or disprove the following.

(a) If there exist $w \in \mathbb{C}$ and $r > 0$ such that there is no $z \in \mathbb{C}$ such that $f(z) \in B(w; r)$, then f is constant.

(b) If there exists $M > 0$ such that $|u(x, y)| \leq M$ for all $z = x + iy \in \mathbb{C}$, then f is constant.

(c) If there exists $M > 0$ such that $|v(x, y)| \leq M$ for all $z = x + iy \in \mathbb{C}$, then f is constant.

Q-2) Prove or disprove the following.

(a) $f(z) = z$ is the only entire function with the propert that $f(n) = n$ for every integer n.

(b) $f(z) = \pi$ is the only entire function with the property that $f(n) = \pi$ for every integer n.

Q-3) Show that for every integer $n \geq 2$, we have

$$\int_0^\infty \frac{dx}{1 + x^n} = \frac{\pi}{n \sin \frac{\pi}{n}}.$$

Q-4) Show that $\cos \pi z = \displaystyle\prod_{n=1}^{\infty} \left(1 - \frac{4z^2}{(2n-1)^2}\right)$.

Q-5) Find a formula for the value of the integrals $I_n = \displaystyle\int_0^{\pi/2} \frac{dx}{1 + \tan^{n!} x}$, where $n \geq 4$ is an integer.

Grading=20 points each