Bilkent University

Name & Lastname: ....

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Scan and save your answer as a pdf file and mail it to me before the deadline.

Q-1) On page 43 of Conway's book we have the theorem:

> **2.30 Theorem.** *Let G be either the whole plane $\mathbb{C}$ or some open disk. If $u\colon G \to \mathbb{R}$ is a harmonic function then u has a harmonic conjugate.*

After the proof the section ends with the following paragraph:

> Where was the fact that G is a disk or $\mathbb{C}$ used? Why can't this method of proof be doctored sufficiently that it holds for general regions G? Where does the proof break down when $G = \mathbb{C} - \{0\}$ and $u(z) = \log |z|$?

Discuss your answers to these questions.