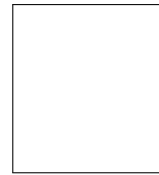




Bilkent University

Homework # 03
Math 503 Complex Analysis I
Due: 3 December 2020 Thursday
Instructor: Ali Sinan Sertöz



Name & Lastname:

Department:

Student ID:

Scan and save your answer as a pdf file and mail it to me before the deadline.

Q-1) Prove the following identities where $a \in \mathbb{C}$ but is not an integer.

$$(a) \frac{\pi^2}{\sin^2 \pi a} = \sum_{n=-\infty}^{\infty} \frac{1}{(a+n)^2}$$

$$(b) \pi^2 = 8 \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$$

$$(c) \pi \cot \pi a = \frac{1}{a} + \sum_{n=1}^{\infty} \frac{2a}{a^2 - n^2}$$

$$(d) \frac{\pi}{\sin \pi a} = \frac{1}{a} + \sum_{n=1}^{\infty} (-1)^n \frac{2a}{a^2 - n^2}$$

Remarks: The result of (c) is crucially used in the factorization of the sine function. All these identities are proved in a very similar manner so they can all be considered as the manifestation of a single idea. All the information needed to attack these identities are explained in detail on page 122 of Conway's book.