Bilkent University

Midterm # 01
Math 503 Complex Analysis I
Due: 18 November 2020
Instructor: Ali Sinan Sertöz

Name & Lastname: ....

Department: ....

Student ID: ....

You may and in fact must discuss your solutions with your friends. Consult any source available. After you ***understand*** the solution start writing your own solution in your own words. When you get stuck, again talk with your friends and consult sources before you continue to write your own solution in your own words.

At the end of each solution quote the sources that you used, including your friends's names who provided you with useful ideas. This is professionalism!

Never borrow your friend's written solution and never lend your written solution.

Then scan and save your solutions as one pdf file and mail it to me before the deadline.

Q-1) Let $f(z) = u(x, y) + iv(x, y)$ be a C^1-function on $\mathbb{C}$. Here as usual u and v are real valued C^1-function of the real variables x and y, and $z = x + iy$. Assume that f is conformal. Show that f is complex analytic.

Q-2) If $f(z)$ is analytic on a region G and is zero on a non-empty open subset U of G, then $f(z) \equiv 0$ on G. This is in stark contrast with what is possible in real analysis. To see this wide difference between these two worlds construct a real valued, non-negative C^∞-function $f(x)$ of the real variable x with the property that $f(x) = 1$ on the open interval $(-1, 1)$, and is zero outside the interval $(-2, 2)$.

Q-3) Let $m \geq 1$ be an integer, and define

$$F(m) = \int_{|z|=1} \frac{\sin z}{z^m}\, dz,$$

where the integration is taken counterclockwise. Find an explicit formula for $F(m)$.

Q-4) Show that

$$\tan z = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{2^{2n}(2^{2n}-1)B_{2n}}{(2n)!}\, z^{2n-1}, \quad |z| < \frac{\pi}{2},$$

where B_n are Bernoulli numbers with the convention that $B_0 = 1$ and $\sum_{k=0}^{n} \binom{n+1}{k} B_k = 0$, for $n \geq 1$.

Grading=25 points each