Due Date: 11 March 2013, Monday NAME:..

Ali Sinan Sertöz STUDENT NO:..

Math 504 Complex Analysis II – Take-Home Exam 01

1	2	3	4	5	TOTAL
25	25	25	25	0	100

Please do not write anything inside the above boxes!

Check that there are **4** questions on your exam booklet. Write your name on top of every page. Show your work in reasonable detail.

For each question I will post the best student solution on the web. If there are more than one interesting solutions, I will post them all. Having your solution posted on the web will get you extra 10 points for each solution posted. These will be added to your total exam grades before an average is taken at the end of the semester.

NAME: STUDENT NO:

Q-1) Show that complex conjugation of Σ corresponds to reflection of S^2 in the plane $x_2 = 0$. What transformation of Σ correspond to the reflections in the plane $x_1 = 0$ and $x_3 = 0$? Show that the antipodal map $Q \mapsto -Q$ of S^2 is the composition of the above three reflections in any order and hence express it as a transformation of Σ.
[page 15, Exercise 1E]

Solution:

NAME: STUDENT NO:

Q-2) Let f be a rational function whose poles in $\mathbb{C}$ are $\beta_1, \ldots, \beta_q$. Prove that there exists unique polynomials $\phi_0, \ldots, \phi_q$ with zero constant term such that

$$f(z) = \phi_0(z) + \sum_{i=1}^{q} \phi_i \left(\frac{1}{z - \beta_i} \right) + \text{constant}.$$

Illustrate this result with reference to the function

$$f(z) = \frac{z^2}{(z-1)^2(z-2)}.$$

[page 16, Exercise 1L]

Solution:

NAME: STUDENT NO:

Q-3) Let $f(z)$ be a rational function such that $|z| = 1$ implies $|f(z)| = 1$. Show that α is a zero of $f(z)$ if and only if $1/\bar{\alpha}$ is a pole of $f(z)$, and hence find the most general form of $f(z)$.
[page 16, Exercise 1M]

Solution:

NAME: STUDENT NO:

Q-4) Investigate the covering of the sphere by the sphere associated with the rational function

$$f(z) = \frac{z^3}{z^4 + 27}.$$

[page 16, Exercise 1N]

Solution: