



Bilkent University

Take-Home Exam # 05  
Math 633 Algebraic Geometry  
Due on: 12 December 2019 Thursday - Class Time  
Instructor: Ali Sinan Sertöz



Name & Lastname: .....

Department: .....

Student ID: .....

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**Q-1)** Hartshorne, Example II.3.0.1, page 82.

After reading the three definitions on page 82 of Hartshorne's book, prove the following statements which are quoted in the above mentioned example without proof. Here  $X = \text{Spec } A$  is an affine scheme.

- (i)  $X$  is irreducible if and only if the nilradical  $\text{nil } A$  of  $A$  is prime.
- (ii)  $X$  is reduced if and only if  $\text{nil } A = 0$ .
- (iii)  $X$  is integral if and only if  $A$  is an integral domain.

**Solution:**