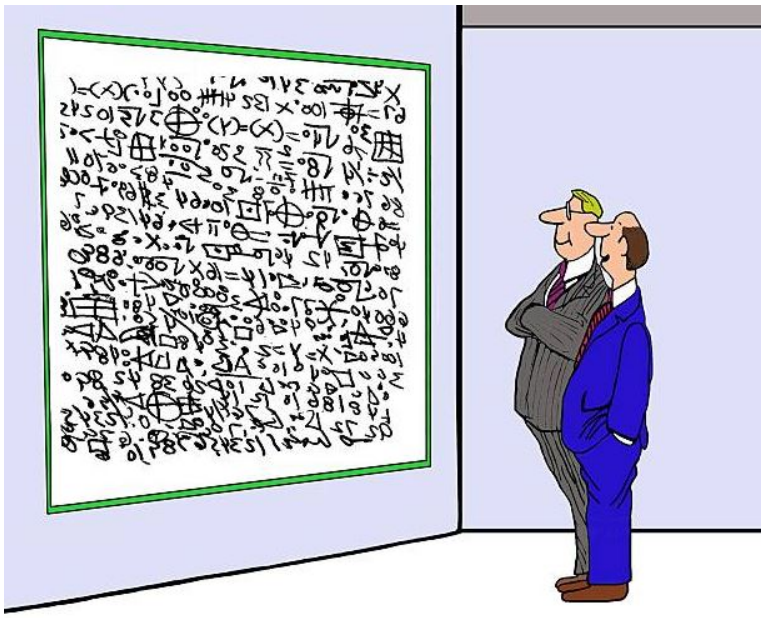
Math 503 Complex Analysis I
Midterm Exam 1
November 6, 2026
10:30 - 12:20

Name :

ID# :

There is a bonus problem at the end which is not assigned any points. If you solve it nicely to demonstrate that you are comfortable with the techniques we learned so far, then I will be more tolerant when I am reading your answers to the exam questions. Your fluency in the solution of the bonus question shows that any mistake you make in the exam solutions is an accident rather than the consequence of lack of knowledge.

Q1	Q2	Q3	Q4	Total
25	25	25	25	100



"When you put it like that, it makes complete sense."

1. In the following simplify your answer as much as possible.

(i) Find the absolute value and conjugate of $\omega = \dfrac{3-i}{\sqrt{2}+3i}$.

(ii) Evaluate the cross-ratio $(2, 1-i, 1, 1+i)$.
Hint: The cross-ratio (z_1, z_2, z_3, z_4) is the image of z_1 under the Möbius transformation which takes z_2 to 1, z_3 to 0, and z_4 to ∞.

(i) is on page 2, (ii) is on page 54.

2. Let f be an entire function and suppose that there is a constant M, an $R > 0$, and an integer $n \geq 1$ such that $|f(z)| \leq M|z|^n$ for all $|z| > R$. Show that f is a polynomial of degree $\leq n$.

Hint: You may consider using Cauchy's Estimate: Let f be analytic in $B(a; R)$ and suppose $|f(z)| \leq M$ for all z in $B(a; R)$. Then $|f^{(n)}(a)| \leq \dfrac{n!M}{R^n}$. (as given on page 73 of Conway's book.)

This is exercise 1 on page 80

3. Let $\lambda > 1$ and show that the equation $\lambda - z - e^{-z} = 0$ has exactly one solution in the half plane $\{z : \text{ Re } z > 0\}$. Show that this solution must be real. What happens to the solution as $\lambda \to 1$?

4. Using the residue theory show that

$$\int_0^\infty \frac{x^2\,dx}{(x^2+\pi^2)^2} = \frac{1}{4\pi^2}.$$

Bonus: Prove the Casorati-Weierstrass Theorem: If f has an essential singularity at $z = a$, then for every $\delta > 0$, the closure of the set $\{\ f(z) \ : \ 0 < |z - a| < \delta\ \}$ is all of $\mathbb{C}$.

Hint: There are different proofs of this result. All proofs start by assuming to the contrary that the result does not hold, and then find a way of showing that a cannot be an essential singularity under that assumption.

Casorati-Weierstrass Theorem is on page 109 of Conway.