

Achievements of our geometry group during
the first thirty years of our department, and a
little bit of history

Ali Sinan Sertöz
Bilkent University
Department of Mathematics
Ankara, Turkey

June 2022



Dedicated to Professor Mefharet Kocatepe
on the occasion of her retirement
with gratitude for her service as the department head for more than two
decades

UNAM
Conference
Hall

CONFERENCE IN HONOR OF
Professor Kocatepe

ON THE OCCASION OF HER RETIREMENT
AND TO CELEBRATE

30 YEARS OF THE
DEPARTMENT OF
MATHEMATICS
OF BILKENT
UNIVERSITY

**JUNE 3-4
2022**

 Bilkent University

Some prehistory

I arrived at Bilkent late at night on 18 January 1988 Monday.

Before that I was a junior researcher at TÜBİTAK's Gebze research institute working with Cahit Arf and brewing deep friendships over coffee with theoretical physicists at the institute. We were living at the institute's lodgings overlooking İzmit Bay. The pay was minimal but as Tevye, in *Fiddler on the Roof* (1971), said about his daughter and his son in law, we were so happy, we didn't know how miserable we were.

That was the panorama when Bilkent entered my life.

I was immediately immersed in teaching. I had some real experience in teaching during the last years of my graduate studies but at Bilkent I was encountering some difficulties. The students were not paying attention. Some of them were openly reading newspapers in class. The others were asking questions which bordered on the line of sarcasm. By ignoring their naughtiness and answering calmly their questions I started to gain their respect but the progress was painfully slow.

Then at the end of the month I received my first Bilkent wage. Tülin and I went shopping and bought me new clothes.

The next day when I entered class all the newspapers were quickly shoved away. There was a respectful silence in class. They only asked technical questions and listened carefully to my explanations. I had gained the respect of the class instantly.

This was the first lesson I learned about life at Bilkent. Of course that lesson had been on the air by Nasrettin Hoca's "*ye kürküm ye*" story for centuries but as they say "*bir musibet bin nasihatten iyidir*".

Meanwhile here there was a mathematics department *de facto* but we were not officially a department. We even had a department head; Yavuz Nutku, a theoretical physicist from Gebze. He was recruited and assigned as the department head. He brought with him Hasan Gümral, his Ph.D. student. We also had a few other graduate students and I remember lecturing to my

heart's content on juicy graduate subjects. Hasan later became our first graduate.

One day when I was baby sitting my son who was a few months old at the time Yavuz Bey phoned me and called me to his office on an urgent departmental business. Being a young father and valuing my son over anything else I of course refused to go. He insisted and even suggested that I pack Emre and bring him along and continue baby sitting him at his office.

So Emre was in his baby bag and was placed near the radiator to keep him warm while Mefharet, Yavuz and I sat together for that urgent business.

The university administration had decided to make the mathematics department official and for this Yavuz Bey had to prepare a curriculum and syllabus for each course. This then would be submitted to the approval of the university senate.

I remember that we sat at a long desk facing a wall. Yavuz in the middle, Mefharet on his right and I on his left. Mefharet and I were leaning forward to face each other to discuss what courses to include in the curriculum. After that we shared the process of writing syllabi. As we wrote a syllabus we were giving it to Yavuz who was cutting off the blank parts of the paper and taping it to the previous texts. This resulted in the long sheet to be given to the secretary to be typed out properly.

Although there were several additions over the years I can still recognize my sentences in some of the syllabi that I authored that day while checking the comfort of my son every few minutes.

My involvement in this process can be recorded as the first contribution of our geometry group to the department.

It is also a sweet coincidence that years later Emre became a student of our department. He completed his bachelor's degree as the department *summa cum laude*, which was not surprising because after all he was there when it all started.



From left to right: Mefharet Kocatepe, Mehmet Kocatepe, Yavuz Nutku, Can Şınga Mugan, Lütfiye Nutku, Uluğ Çapar, Ali Sinan Sertöz, Uğurhan Mugan with Emre Can Sertöz on his shoulders, and Can Delale. The date is circa 1990, location is most likely the Bilkent lodging of Yavuz Nutku.

Our first undergraduate students

In the fall of 1991 we had our first undergraduate students. Most of them went to English preparatory school and we had only three students in our first program. They were Ali Bıçak, Semail Ülgen and İsmail Nazlı.

Ali Bıçak was a brilliant mathematics student. He was my first senior project student. I still remember the comfort and self confidence with which he talked about the Riemann-Roch theorem and the Riemann-Hurwitz formula during his senior project presentations. He later went to ODTÜ to have a masters degree in cryptology. He received a PhD at Maryland University. Later he worked for an IT company for some time until he joined Marymount

University in Virginia where he is now.

Semail Ülgen received her PhD degree at Purdue University and is now at Antalya Bilim University.

İsmail Nazlı was never a devoted mathematics student and we lost track of him after graduation. Lately I found him on Facebook. He lives in Toronto and apparently works as a manager in a firm.

We never hit this 66% success rate later!

Bilkent Distinguished Teaching Awards

As a department we value the importance of teaching and passing the torch to young generations. In this vein our department has won six distinguished teaching awards, two of which belong to our geometry group. Klyachko received this award in 1999, and I received it in 2007.

It is worth mentioning the names of the remaining four recipients: Mefharet Kocatepe in 1997, Iossif Ostrovskii in 1998, Murat Alkar in 2001 and Azer Kerimov in 2003.

Sedat Simavi Science Awards

Mefharet Kocatepe and Metin Gürses received their well deserved Sedat Simavi Science Awards a few years prior to their coming to Bilkent. Three fulltime members of our department later received this award and one of them again belongs to our geometry group. I received this award in 2005, Müfit Sezer in 2011 and Hamza Yeşilyurt in 2014.

Our geometry group

I have listed in the references all the articles published by members of our department with the primary or secondary mathematics subject classification

14 Algebraic Geometry.

Algebraic geometry can be considered as a branch of pure mathematics where geometers explore the meaning and interrelation of some objects which they call geometric. On the other hand some algebraic geometers try to find structures in algebraic geometry which will be used to formulate and explain all other branches of mathematics. This imperialistic quest so far succeeded only to invade certain parts of number theory which is now called arithmetic algebraic geometry. In this respect I want to quote David Mumford who received Fields Medal in 1974 for his work in algebraic geometry. Later when his wife Erika died because a tumor in her body had not been detected on time, Mumford turned to pattern recognition. He later talked about his impressions about algebraic geometry and algebraic geometers in a famous interview. He said:

Algebraic geometry seems to have acquired the reputation of being esoteric, exclusive, and very abstract, with adherents who are secretly plotting to take over all the rest of mathematics. In one respect this last point is accurate.

On a more humble note you can see algebraic geometry as a collection of esoteric tools to solve otherwise untractable mathematical problems. Our group members certainly adhere to different points of view at different times in their works.

I will now briefly talk about some individual members of our group and their contributions.

Sergei Stepanov



Here we are at a coffee break during our Algebraic Geometry Summer School of 1995. I stand on the left end and Stepanov, with sunglasses, on the right end.

Stepanov was at Bilkent between 28.09.1993 and 30.01.2004.

Stepanov was not a geometer *per se*; he was working on coding theory but most of his codes were on elliptic curves and elliptic curves are our beloved geometric objects so he was always considered to be a member of our group. He was an arithmetic algebraic geometer after all.

He was a celebrity before he came to Bilkent. He already contributed immensely to coding theory with a state medal to his name. At Bilkent his major contribution was a graduate student, Ferruh Özbudak, who is now a renowned mathematician in his field and is working at ODTÜ.

Stepanov was my office neighbor. When we talked about mathematics he would speak with a fluent English and would understand me even when my English sometimes failed me. But when I approached him for a bureaucratic business he would play deaf and mute. That is how he avoided all depart-

mental chores. However after a while we built a mutual trust and friendship and when I needed his signature for some papers I would just ask him to sign where I pointed. He would sign without a word and quickly return to his calculations. His dislike for paperwork was such that he did not even bother to write reports on some promotion dossiers that were sent to him and trusting our friendship he would ask me to write those reports which he would sign and immediately send. This way I wrote several professorship promotion reports on Stepanov's behalf while I was still an assistant professor.

Stepanov was also fond of dogs. His dogs were ugly but precious and were occasionally stolen. He would go and buy an even uglier dog which was more precious and would be stolen more quickly. Sometimes after a dog is stolen I would tease him that it was ugly anyway but he would in a saddened voice object my using the pronoun *it* and correct me by saying *she*.

Stepanov was also a heavy pipe smoker. He could not deal with his codes without smoking. I always appreciated his good taste in choosing tobacco and when he eventually left Bilkent and returned to Moscow I suffered withdrawal symptoms of tobacco smoke for a long time.

Alexander Klyachko



Klyachko was at Bilkent between 02.10.1995 and 31.10.2019.

Klyachko was one of those mathematicians who knew almost everything. He considered algebraic geometry only as a collection of tools to solve important mathematics problems. His work was deep and meaningful. One of the very first articles he wrote, *Lie elements in a tensor algebra* in 1974, is still getting citations today.

When at Bilkent he even collaborated with Shumovski from the physics department to write an article on entanglement. For some time he worked with his student Murat Altunbulak on Pauli principle.

But Klyachko's real fame came when in 1998 he published a solution to the problem of finding the eigenvalues of the sum of two hermitian matrices in terms of the eigenvalues of the summand matrices, see [68].

His solution was immediately exposed by William Fulton in a Bourbaki Sem-

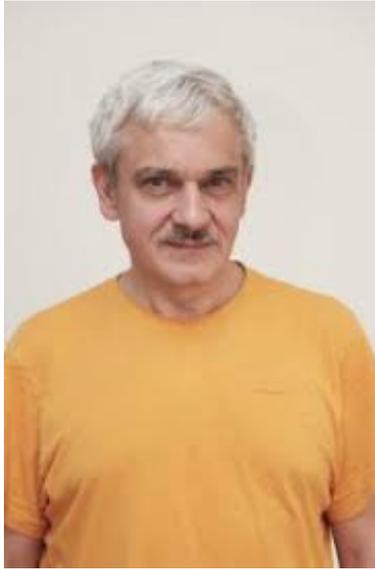
inar that year, no: 845. In 2000 Fulton even wrote a more detailed exposition in the Bulletin of the American Mathematical Society.

For this result Klyachko was an invited speaker of the International Congress of Mathematician in 2002 in Beijing. When on the morning of 28 August 2002 he gave his talk titled "*Vector bundles, linear representations, quantum cohomology, and spectral problems*" he became the first invited speaker from Turkey. You can find a copy of his talk in the Congress proceedings, [65].



Klyachko enjoying a leisure time. His T-shirt reads "Your village called, their idiot is missing". He could afford such a joke!

Alexander Degtyarev



Degtyarev came to Bilkent on 01.10.1996.

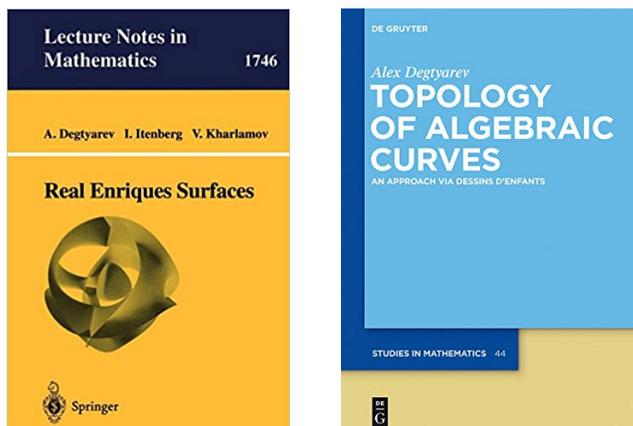
Alex is the most productive member of our group. He has a way of combining algebraic geometry and algebraic topology techniques in attacking and solving problems. Among his numerous achievements I will mention, in arbitrary order, only a few, due to my lack of expertise to expose them properly.

In 2015 in his article "*Lines generate the Picard groups of certain Fermat surfaces*", see [41], he solved a long standing problem about the generators of these Picard groups.

In 2017 there was a joint article of Alex with me and Ilia Itenberg, see [51]. In my observation he was the major contributor. There again a long standing question was affirmatively solved. Since Torelli theorem holds for K3 surfaces, all the geometric questions about a K3 surface should be answered by studying the associated K3 lattice. In particular the maximal number of lines that can be seen on a K3 surface is found to be 64 by using elliptic fibration techniques by Schütt and Rams, after the pioneering work of Segre. This result still had to be obtained through lattice theoretical means. This article solves that problem thoroughly and even provides information about

the possible number of lines on a K3 surface. This last result was not obtained through other techniques.

Alex continued his research on the number of lines on surfaces with several other publications which are not yet all published. one such result however is [43].



He published two books. The first one is joint with Itenberg and Kharlamov, titled "Real Enriques Surfaces" published in 2000 by Springer Verlag in the Lecture Notes in Mathematics series as volume 1746. The book describes the topology of real Enriques surfaces and classifies them up to deformation equivalence, [12].

His second book is a solo work and is published in 2012 as De Gruyter Studies in Mathematics volume 44 and is titled "Topology of Algebraic Curves: An Approach via dessin d'enfants", [34]. Later when I organized a seminar based on this book he humbly joined by saying "It is time I finally learn what this is all about."

I am not capable of doing any further justice to his accomplishments. I will leave the impressive list of his publications to speak on his behalf.

Ali Sinan Sertöz

When we were graduate students coming up with our first results we would apologetically explain our results to our professors and we would say that it is a small lemma or may be just an observation. Then we would say may be we should not even have brought it to their office and take their time with this triviality. Our professors would smile and tell us that we should never downplay or depreciate our own work. Our friends would do a much better job at that!

Therefore here I talk about my contributions as a member of the geometry group, unabashed and in detail.

When I first came to Bilkent I was still young and foolish, present company excepted. I thought I could and *must* change the world.

One of my main worries was the reasons behind the destruction of the ancient library of Alexandria. We will probably never find out the actual culprit. Either an angry mob who was ready to burn down any building of authority destroyed the library or while the library was being destroyed for whatever reason the public did not find it necessary to salvage its belongings.

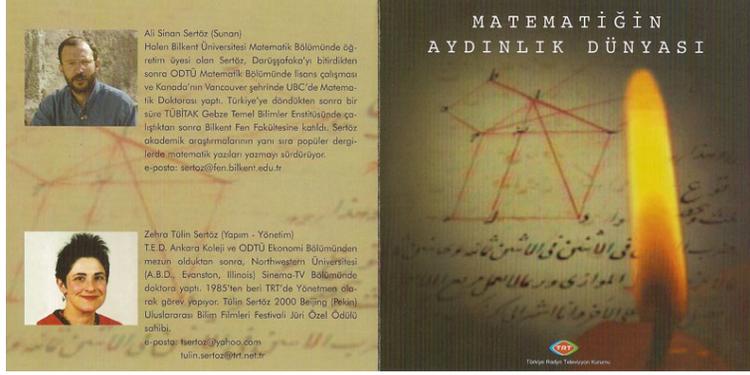
The more I thought about it the more I was convinced that doing pure research oblivion to the public is an elite and sterile act. Sooner or later the public will either burn down your ivory tower or start questioning the share you receive from taxes.

The public starts to see scientists as arrogant and snobbish. And the scientists look down on anyone who tries to outreach to the public.

The problem about this gap had to be addressed.

I probably talked too much about this at home that Tülin eventually decided to shoot a four part documentary on mathematics. That is how "*Matematiğin Aydınlık Dünyası*" came about. Her documentary was broadcast several times on TRT and later when RTÜK gave punishment to some private TV stations they broadcast Tülin's documentary at the hour of their banned programs. That was most of the time the prime time of television. Thus her documentary reached literary to millions.

If our department has not yet been burned down we owe it to her!



DVD cover of Matematığın Aydınlık Dünyası

Later I wrote a book based on this documentary which to this day has sold above one hundred thousand copies, and we are still counting.

In 1995 I organized a summer school on algebraic geometry. The school hosted 61 participants from twelve different countries and Turkey. The speakers were celebrities in algebraic geometry. Hocabey invited the speakers to his mansion. Two of our speakers turned out to be amateur singers and they performed at Hocabey's house after dinner. The proceedings were refereed and published by Marcel Dekker, [80].

I was hoping that this school would start a wave of activities in algebraic geometry in Turkey. It did not.

**SUMMER SCHOOL
ON ALGEBRAIC GEOMETRY
CEBİRSEL GEOMETRİ
YAZ OKULU
7-19 August 1995**

The summer school is intended for all geometers, graduate students and researchers. Accomodations will be provided by the school through the generous contributions of TÜBİTAK and Bilkent University.
Application deadline: June 15, 1995
*Yaz okulu geometri alanında çalışan tüm araştırmacı ve lisans üstü öğrencilere yöneliktir. Konaklamaları yemek ve yatacağı yer masraflarını TÜBİTAK ve Bilkent Üniversitesinin katkılarıyla karşılanacaktır.
Son başvuru tarihi: 15 Haziran 1995

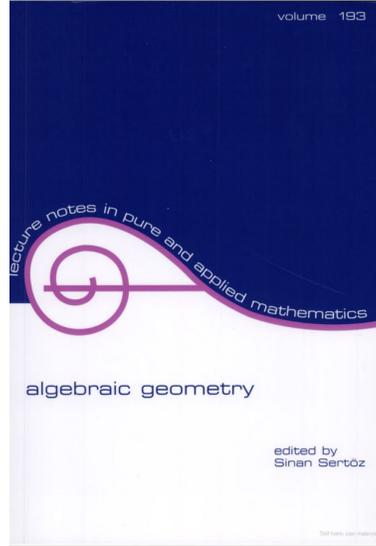


Addresses for applications and inquiries:
Başvuru adresleri:
mail: BICASS, Bilkent University,
06533 Bilkent, Ankara, Turkey
e-mail: bicass@fen.bilkent.edu.tr or
bicass3@bilun.bilnet
WWW: <http://www.fen.bilkent.edu.tr/~bicass3/>
phone: 90-312-266 4377
fax: 90-312-266 4579

Organizing Committee
Organizasyon Komitesi
Sinan Bertoz (MITL)
Hüseyin Çeliker (METU)
Osman Tekenen (Bilkent)
Yıldırım Özen (METU)

BICASS • BILKENT INTERNATIONAL CENTER FOR ADVANCED STUDIES • BILKENT İLERİ ÇALIŞMALAR MERKEZİ • BILKENT UNIVERSITY ANKARA/TURKEY

List of lecturers:
Konuşmacılar (Listed):
D. Eisenbud (SMA, France)
D. Eisenbud (SMA, France)
D. Eisenbud (SMA, France)
D. Eisenbud (SMA, France)
D. Eisenbud (SMA, France)
D. Eisenbud (SMA, France)
D. Eisenbud (SMA, France)
D. Eisenbud (SMA, France)
D. Eisenbud (SMA, France)
D. Eisenbud (SMA, France)



However not accepting defeat I collaborated with my friends at ODTÜ to start a seminar series on algebraic geometry. The first talk of ODTÜ-Bilkent Algebraic Geometry Seminars was given on 29 September 2000 Friday at 15:40. At the end of this semester we closed the season with our 511th talk. Lately we went online and we are having speakers and attendants from all over the world. In particular at our last talks we had attendance from China and California. You can hear us on both sides of the Pacific!

During one of these seminars I was bored; meaning that I failed to follow the speaker whose topic was well beyond my capacity. I always carry an article or two for such occasions. I started to read an article of Keum on $K3$ surfaces. Some of his calculations looked unnecessarily complicated to me and I started to scribble my easy approach in the margins. One last example resisted all my attacks during and after the seminar.

I got obsessed with that case and kept working on it for some time. Then one day Degtyarev suggested that I read a few papers of Vinberg who was dealing

with similar equations but in a totally different setting. There I found how to overcome my difficulties. I wrote an article on my findings. It was published in the Proceedings of the American Mathematical Society, see [76]. And that is how I got my Sedat Simavi Science Award.

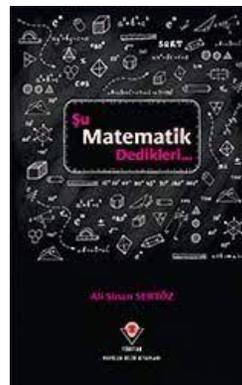
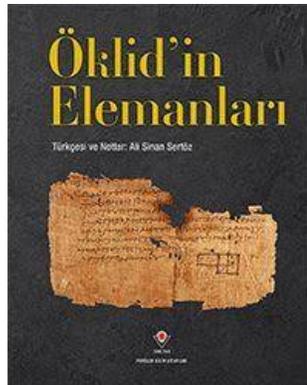
Meanwhile I had the opportunity of writing popular science articles in TÜBİTAK's *Bilim ve Teknik* journal which has a readership around thirty thousand each month. My articles were lately collected and published as a book titled "*Şu Matematik Dedikleri*".

When the book came out nothing happened. No fireworks, no concerts around the country, not even book signing sessions.

That is when I started to accept that I cannot change the world.

Then I decided to do something selfish. I studied Euclid's *Elements* and translated it to Turkish just for my personal use. One thing lead to another and TÜBİTAK published this translation. It is the first Turkish translation of *Elements* into Turkish since Euclid wrote it on scrolls about 2300 years ago.

We may count this honor also as a contribution of our geometry group to our department.



Meanwhile our department

I am a pure mathematician to a fault, I must admit. However I am very happy that we are recently hiring highly talented young applied mathematicians to our department. They are enriching the scientific environment of the department.

I was hoping that I would watch them ride the department to new horizons while I prepare to take my silent exit from the scene, now that I know I cannot change anything.

But lately I found myself sitting on juries for senior projects of applied mathematics students and getting excited about what they are doing.

This is a sign that the inevitable is happening; the world is changing me!

In particular Gökhan Yıldırım, one of our distinguished young applied mathematicians, kindly sent me a few articles where these applied mathematicians are using Schubert calculus to attack some of their problems. Schubert calculus is actually one of the purest topics of pure geometry and I did have one or two articles on that topic when I was young. Now I don't know if I should follow suit and start recalling what I knew about Schubert Calculus and learn ways of implementing that on some problems of applied mathematics.

Things are changing and one cannot help but ask what the future will bring to our department.

If not the answer but a lead to ponder on that question lies in Alfred Hitchcock's 1956 thriller *The Man Who Knew Too Much*. There Doris Day sings with her sensual and yet strong voice

Que sera, sera
Whatever will be, will be
The future's not ours to see

The references below list the articles or books published by members of our department with the primary or secondary Mathematics Subject Classification 14 Algebraic Geometry, as of June 2022.

References

- [1] Ayşegül Akyol. Classical Zariski pairs. *J. Knot Theory Ramifications*, 21(9):1250091, 16, 2012.
- [2] Ayşegül Akyol and Alex Degtyarev. Geography of irreducible plane sextics. *Proc. Lond. Math. Soc. (3)*, 111(6):1307–1337, 2015.
- [3] Feza Arslan. Cohen-Macaulayness of tangent cones. *Proc. Amer. Math. Soc.*, 128(8):2243–2251, 2000.
- [4] Feza Arslan and Nil Şahin. A fast algorithm for constructing Arf closure and a conjecture. *J. Algebra*, 417:148–160, 2014.
- [5] Feza Arslan, Anargyros Katsabekis, and Melissa Nalbandiyan. On the Cohen-Macaulayness of tangent cones of monomial curves in $\mathbb{A}^4(K)$. *Turkish J. Math.*, 43(3):1425–1446, 2019.
- [6] Feza Arslan and Sinan Sertöz. Genus calculations of complete intersections. *Comm. Algebra*, 26(8):2463–2471, 1998.
- [7] Erwan Brugallé, Alex Degtyarev, Ilia Itenberg, and Frédéric Mangolte. Real algebraic curves with large finite number of real points. *Eur. J. Math.*, 5(3):686–711, 2019.
- [8] Mesut Şahin and Nil Şahin. On pseudo symmetric monomial curves. *Comm. Algebra*, 46(6):2561–2573, 2018.
- [9] Mesut Şahin and Nil Şahin. Betti numbers for certain Cohen-Macaulay tangent cones. *Bull. Aust. Math. Soc.*, 99(1):68–77, 2019.
- [10] Nil Şahin. 4-generated pseudo symmetric monomial curves with not Cohen-Macaulay tangent cones. *Turkish J. Math.*, 44(6):2174–2184, 2020.

- [11] Berrin Şentürk and Özgün Ünlü. Carlsson’s rank conjecture and a conjecture on square-zero upper triangular matrices. *J. Pure Appl. Algebra*, 223(6):2562–2584, 2019.
- [12] A. Degtyarev, I. Itenberg, and V. Kharlamov. *Real Enriques surfaces*, volume 1746 of *Lecture Notes in Mathematics*. Springer-Verlag, Berlin, 2000.
- [13] A. I. Degtyarev. Quintics in \mathbf{CP}^2 with nonabelian fundamental group. *Algebra i Analiz*, 11(5):130–151, 1999.
- [14] A. I. Degtyarev. A divisibility theorem for the Alexander polynomial of a plane algebraic curve. *Zap. Nauchn. Sem. S.-Peterburg. Otdel. Mat. Inst. Steklov. (POMI)*, 280(Geom. i Topol. 7):146–156, 300, 2001.
- [15] A. I. Degtyarev and V. M. Kharlamov. Topological properties of real algebraic varieties: Rokhlin’s way. *Uspekhi Mat. Nauk*, 55(4(334)):129–212, 2000.
- [16] A. I. Degtyarev and V. I. Zvonilov. Rigid isotopy classification of real algebraic curves of bidegree $(3, 3)$ on quadrics. *Mat. Zametki*, 66(6):810–815, 1999.
- [17] Alex Degtyarev. Fundamental groups of symmetric sextics. *J. Math. Kyoto Univ.*, 48(4):765–792, 2008.
- [18] Alex Degtyarev. Oka’s conjecture on irreducible plane sextics. *J. Lond. Math. Soc. (2)*, 78(2):329–351, 2008.
- [19] Alex Degtyarev. On deformations of singular plane sextics. *J. Algebraic Geom.*, 17(1):101–135, 2008.
- [20] Alex Degtyarev. Stable symmetries of plane sextics. *Geom. Dedicata*, 137:199–218, 2008.
- [21] Alex Degtyarev. Fundamental groups of symmetric sextics. II. *Proc. Lond. Math. Soc. (3)*, 99(2):353–385, 2009.
- [22] Alex Degtyarev. Irreducible plane sextics with large fundamental groups. *J. Math. Soc. Japan*, 61(4):1131–1169, 2009.

- [23] Alex Degtyarev. Oka’s conjecture on irreducible plane sextics. II. *J. Knot Theory Ramifications*, 18(8):1065–1080, 2009.
- [24] Alex Degtyarev. On irreducible sextics with non-abelian fundamental group. In *Singularities—Niigata–Toyama 2007*, volume 56 of *Adv. Stud. Pure Math.*, pages 65–91. Math. Soc. Japan, Tokyo, 2009.
- [25] Alex Degtyarev. Zariski k -plets via dessins d’enfants. *Comment. Math. Helv.*, 84(3):639–671, 2009.
- [26] Alex Degtyarev. Classical Zariski pairs. *J. Singul.*, 2:51–55, 2010.
- [27] Alex Degtyarev. Plane sextics via dessins d’enfants. *Geom. Topol.*, 14(1):393–433, 2010.
- [28] Alex Degtyarev. Plane sextics with a type E_8 singular point. *Tohoku Math. J. (2)*, 62(3):329–355, 2010.
- [29] Alex Degtyarev. The fundamental group of a generalized trigonal curve. *Osaka J. Math.*, 48(3):749–782, 2011.
- [30] Alex Degtyarev. Hurwitz equivalence of braid monodromies and extremal elliptic surfaces. *Proc. Lond. Math. Soc. (3)*, 103(6):1083–1120, 2011.
- [31] Alex Degtyarev. Plane sextics with a type- E_6 singular point. *Michigan Math. J.*, 60(2):243–269, 2011.
- [32] Alex Degtyarev. Topology of plane algebraic curves: the algebraic approach. In *Topology of algebraic varieties and singularities*, volume 538 of *Contemp. Math.*, pages 137–161. Amer. Math. Soc., Providence, RI, 2011.
- [33] Alex Degtyarev. Dihedral coverings of trigonal curves. *Indiana Univ. Math. J.*, 61(3):901–938, 2012.
- [34] Alex Degtyarev. *Topology of algebraic curves*, volume 44 of *De Gruyter Studies in Mathematics*. Walter de Gruyter & Co., Berlin, 2012. An approach via dessins d’enfants.

- [35] Alex Degtyarev. Toward a generalized Shapiro and Shapiro conjecture. In *Perspectives in analysis, geometry, and topology*, volume 296 of *Progr. Math.*, pages 67–79. Birkhäuser/Springer, New York, 2012.
- [36] Alex Degtyarev. Transcendental lattice of an extremal elliptic surface. *J. Algebraic Geom.*, 21(3):413–444, 2012.
- [37] Alex Degtyarev. On plane sextics with double singular points. *Pacific J. Math.*, 265(2):327–348, 2013.
- [38] Alex Degtyarev. The Alexander module of a trigonal curve. *Rev. Mat. Iberoam.*, 30(1):25–64, 2014.
- [39] Alex Degtyarev. On the Artal-Carmona-Cogolludo construction. *J. Knot Theory Ramifications*, 23(5):1450028, 35, 2014.
- [40] Alex Degtyarev. The Alexander module of a trigonal curve. II. In *Singularities in geometry and topology 2011*, volume 66 of *Adv. Stud. Pure Math.*, pages 47–69. Math. Soc. Japan, Tokyo, 2015.
- [41] Alex Degtyarev. Lines generate the Picard groups of certain Fermat surfaces. *J. Number Theory*, 147:454–477, 2015.
- [42] Alex Degtyarev. On the Néron-Severi lattice of a Delsarte surface. *Kyoto J. Math.*, 56(3):611–632, 2016.
- [43] Alex Degtyarev. Lines on smooth polarized $K3$ -surfaces. *Discrete Comput. Geom.*, 62(3):601–648, 2019.
- [44] Alex Degtyarev. Smooth models of singular $K3$ -surfaces. *Rev. Mat. Iberoam.*, 35(1):125–172, 2019.
- [45] Alex Degtyarev. 800 conics on a smooth quartic surface. *J. Pure Appl. Algebra*, 226(10):Paper No. 107077, 5, 2022.
- [46] Alex Degtyarev, Torsten Ekedahl, Ilia Itenberg, Boris Shapiro, and Michael Shapiro. On total reality of meromorphic functions. *Ann. Inst. Fourier (Grenoble)*, 57(6):2015–2030, 2007.
- [47] Alex Degtyarev and Ilia Itenberg. On real determinantal quartics. In *Proceedings of the Gökova Geometry-Topology Conference 2010*, pages 110–128. Int. Press, Somerville, MA, 2011.

- [48] Alex Degtyarev, Ilia Itenberg, and Viatcheslav Kharlamov. Finiteness and quasi-simplicity for symmetric $K3$ -surfaces. *Duke Math. J.*, 122(1):1–49, 2004.
- [49] Alex Degtyarev, Ilia Itenberg, and Viatcheslav Kharlamov. On deformation types of real elliptic surfaces. *Amer. J. Math.*, 130(6):1561–1627, 2008.
- [50] Alex Degtyarev, Ilia Itenberg, and Viatcheslav Kharlamov. On the number of components of a complete intersection of real quadrics. In *Perspectives in analysis, geometry, and topology*, volume 296 of *Progr. Math.*, pages 81–107. Birkhäuser/Springer, New York, 2012.
- [51] Alex Degtyarev, Ilia Itenberg, and Ali Sinan Sertöz. Lines on quartic surfaces. *Math. Ann.*, 368(1-2):753–809, 2017.
- [52] Alex Degtyarev, Ilia Itenberg, and Victor Zvonilov. Real trigonal curves and real elliptic surfaces of type I. *J. Reine Angew. Math.*, 686:221–246, 2014.
- [53] Alex Degtyarev and Viatcheslav Kharlamov. Real rational surfaces are quasi-simple. *J. Reine Angew. Math.*, 551:87–99, 2002.
- [54] Alex Degtyarev and Mutsuo Oka. A plane sextic with finite fundamental group. In *Singularities—Niigata–Toyama 2007*, volume 56 of *Adv. Stud. Pure Math.*, pages 93–108. Math. Soc. Japan, Tokyo, 2009.
- [55] Alex Degtyarev and Nermin Salepci. Products of pairs of Dehn twists and maximal real Lefschetz fibrations. *Nagoya Math. J.*, 210:83–132, 2013.
- [56] Alex Degtyarev and Ichiro Shimada. On the topology of projective subspaces in complex Fermat varieties. *J. Math. Soc. Japan*, 68(3):975–996, 2016.
- [57] Alexander Degtyarev. On the Pontryagin-Viro form. In *Topology, ergodic theory, real algebraic geometry*, volume 202 of *Amer. Math. Soc. Transl. Ser. 2*, pages 71–94. Amer. Math. Soc., Providence, RI, 2001.
- [58] Alexander Degtyarev and Viatcheslav Kharlamov. Real Enriques surfaces without real points and Enriques-Einstein-Hitchin 4-manifolds. In

- The Arnoldfest (Toronto, ON, 1997)*, volume 24 of *Fields Inst. Commun.*, pages 131–140. Amer. Math. Soc., Providence, RI, 1999.
- [59] Sultan Erdoğan Demir. Monodromy groups of real Enriques surfaces. *Topology Appl.*, 159(10-11):2580–2591, 2012.
- [60] Çisem Güneş Aktaş. Classification of simple quartics up to equisingular deformation. *Hiroshima Math. J.*, 47(1):87–112, 2017.
- [61] Koray Karabina, Edward Knapp, and Alfred Menezes. Generalizations of Verheul’s theorem to asymmetric pairings. *Adv. Math. Commun.*, 7(1):103–111, 2013.
- [62] Anargyros Katsabekis. Complete intersection monomial curves and the Cohen-Macaulayness of their tangent cones. *Algebra Colloq.*, 26(4):629–642, 2019.
- [63] Anargyros Katsabekis. Hilbert series of tangent cones for Gorenstein monomial curves in $\mathbb{A}^4(K)$. *Turkish J. Math.*, 45(1):597–616, 2021.
- [64] A. Klyachko and E. Kurtaran. Some identities and asymptotics for characters of the symmetric group. *J. Algebra*, 206(2):413–437, 1998.
- [65] Alexander Klyachko. Vector bundles, linear representations, and spectral problems. In *Proceedings of the International Congress of Mathematicians, Vol. II (Beijing, 2002)*, pages 599–613. Higher Ed. Press, Beijing, 2002.
- [66] Alexander Klyachko. Dynamical symmetry approach to entanglement. In *Physics and theoretical computer science*, volume 7 of *NATO Secur. Sci. Ser. D Inf. Commun. Secur.*, pages 25–54. IOS, Amsterdam, 2007.
- [67] Alexander Klyachko and Orhun Kara. Singularities of the modular curve. *Finite Fields Appl.*, 7(3):415–420, 2001.
- [68] Alexander A. Klyachko. Stable bundles, representation theory and Hermitian operators. *Selecta Math. (N.S.)*, 4(3):419–445, 1998.
- [69] Caner Koca and Ali Sinan Sertöz. Irreducible Heegner divisors in the period space of Enriques surfaces. *Internat. J. Math.*, 19(2):209–215, 2008.

- [70] James D. Lewis and Ali Sinan Sertöz. Motives of some Fano varieties. *Math. Z.*, 261(3):531–544, 2009.
- [71] Huishi Li and Freddy Van Oystaeyen. *A primer of algebraic geometry*, volume 227 of *Monographs and Textbooks in Pure and Applied Mathematics*. Marcel Dekker, Inc., New York, 2000. Constructive computational methods.
- [72] Shu Nakamura. On the classification of the third reduction with a spectral value condition. *J. Math. Soc. Japan*, 49(4):633–646, 1997.
- [73] H. Önsiper and S. Sertöz. On generalized Shioda-Inose structures. *Turkish J. Math.*, 23(4):575–578, 1999.
- [74] Hurşit Önsiper and Sinan Sertöz. On degenerations of fiber spaces of curves of genus ≥ 2 . *Arch. Math. (Basel)*, 69(4):350–352, 1997.
- [75] Hurşit Önsiper and Sinan Sertöz. Generalized Shioda-Inose structures on $K3$ surfaces. *Manuscripta Math.*, 98(4):491–495, 1999.
- [76] Ali Sinan Sertöz. Which singular $K3$ surfaces cover an Enriques surface. *Proc. Amer. Math. Soc.*, 133(1):43–50, 2005.
- [77] Sinan Sertöz. Residues of singular holomorphic foliations. *Compositio Math.*, 70(3):227–243, 1989.
- [78] Sinan Sertöz. A triple intersection theorem for the varieties $SO(n)/P_d$. *Fund. Math.*, 142(3):201–220, 1993.
- [79] Sinan Sertöz. Arf rings and characters. *Note Mat.*, 14(2):251–261 (1997), 1994.
- [80] Sinan Sertöz, editor. *Algebraic geometry*, volume 193 of *Lecture Notes in Pure and Applied Mathematics*. Marcel Dekker, Inc., New York, 1997.
- [81] Sinan Sertöz. An overview of the search for minimal models of algebraic threefolds. In *Algebraic geometry (Ankara, 1995)*, volume 193 of *Lecture Notes in Pure and Appl. Math.*, pages 291–311. Dekker, New York, 1997.
- [82] Sinan Sertöz. Pieri-type intersection formulas and primary obstructions for decomposing 2-forms. *Colloq. Math.*, 87(2):201–210, 2001.

- [83] Serguei A. Stepanov. Character sums, algebraic curves and Goppa codes. In *Algebraic geometry (Ankara, 1995)*, volume 193 of *Lecture Notes in Pure and Appl. Math.*, pages 313–345. Dekker, New York, 1997.
- [84] Serguei A. Stepanov. *Codes on algebraic curves*. Kluwer Academic/Plenum Publishers, New York, 1999.
- [85] İnan Utku Türkmen. Regulator indecomposable cycles on a product of elliptic curves. *Canad. Math. Bull.*, 56(3):640–646, 2013.
- [86] Melih Üçer. On the Alexander invariants of trigonal curves. *Rev. Mat. Complut.*, 35(1):265–286, 2022.